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The Poincaré model of rotation for the synchronous satellites

B. Noyelles
NAmur Center for Complex SYStems (NAXYS), FUNDP - The University of Namur, Belgium & IMCCE, Paris Observatory, France (benoit.noyelles@fundp.ac.be)

Abstract

This communication aims at exploring the possible behaviors of the rotational dynamics of a body locked in 1:1 spin-orbit resonance, consisting of a rigid mantle and a triaxial fluid core, in the approximation of the Poincaré model. For that we perform an extensive numerical study of the system in considering different possible parameters for the size and shape of the core.

We find a classical equilibrium, that can be considered as an extension of the Cassini State 1, around which the influence of the size and polar flattening of the core are highlighted. Moreover, we find a condition for the instability of this equilibrium, that can result in a shift of several degrees of the orientation of the figure axis with respect to the angular momentum.

1 Introduction

In the light of the Galileo and Cassini data, we have a better knowledge of the internal structure and the dynamics of the main natural satellites of Jupiter and Saturne. Most of them are assumed to be differentiated. These data and the future space missions motivate the consideration of an elaborated model of interior to study the rotational dynamics of these bodies.

These bodies have the peculiarity to be locked in 1:1 spin-orbit resonance, what allows to make a general study of their dynamics with respect to the interior parameters. We here focus on the Poincaré-Hough model [2],[4], that considers a rigid mantle and a triaxial fluid core.

2 The model

This model, originally built independently by Hough and Poincaré, put in Hamiltonian form by Touma & Wisdom [5], and applied to Io by Henrard [1], consists to consider a rigid mantle and a triaxial cavity filled with an inviscid fluid of constant uniform density and vorticity. This 4 d.o.f. model considers the longitudinal motion, the obliquity, the polar motion and the pressure coupling at the core-mantle boundary.

We wrote this model in considering 5 interior parameters, i.e.:

- $\epsilon_1 = \frac{2C-A-B}{2C} = J_2\frac{MR^2}{C}$,
- $\epsilon_2 = \frac{B-A}{2C} = 2C_{22}\frac{MR^2}{C}$,
- $\epsilon_3 = \frac{2C_c-A_c-B_c}{2C_c}$,
- $\epsilon_4 = \frac{B_c-A_c}{2C_c}$,
- $\delta = \frac{C_c}{C}$,

A, B and C being the moments of inertia of the whole body, and A_c, B_c and C_c the ones of the core.

The study is a numerical study of the behavior of the system for different possible values of the interior parameters ϵ_3 (polar flattening of the core), ϵ_4 (equatorial ellipticity of the core) and δ (size of the core), consisting in numerical integrations of the Hamilton equations of the system, the initial conditions being chosen as close to the equilibrium as possible.

The variables are derived from the canonical modified Andoyer variables, very similar to the ones of Henrard [1], i.e.

$$\begin{array}{ll} p, & P, \\ r, & R, \\ \xi_1, & \eta_1, \\ \xi_2, & \eta_2, \end{array} \tag{1}$$

the first 3 d.o.f. being related to the orientation of the whole body, and the last one to the velocity field of the fluid filling the core. We have in particular:

- $K = \arccos\left(1 - \frac{R}{P}\right)$ (obliquity of the body),
- $J = \arccos\left(1 - \frac{\xi_1^2+\eta_1^2}{2P}\right)$ (amplitude of the polar motion),
- $J_c = \arccos\left(\frac{\xi_2^2+\eta_2^2}{2\delta P} - 1\right)$ (amplitude of the misalignment of the fluid).

All the numerical simulations have been performed in considering an Io-like body, i.e. with $n = 1297.2$ rad/y, $J_2 = 1.828 \times 10^{-3}$, $C_{22} = 5.537 \times 10^{-4}$, $C/(MR^2) = 0.377$, with a circular orbit or with an eccentricity $e = 4.15 \times 10^{-3}$.

3 Classical behavior

The expected behavior is an extension of Cassini State 1, i.e. P close to unity, J and J_c close to 0, and a small obliquity proportional to the regression rate of the orbital ascending node. The departures from this simplified equilibrium are signatures of the internal structure of the body. One possibility is to study the forced longitudinal librations, that can be observed in some cases, another one is to estimate the periods of the free librations. These librations are expected to have negligible amplitudes, but their periods are a characterization of the way the system reacts to every sinusoidal sollicitations. So, it is of interest to know them.

We have in particular, for the free longitudinal librations u:

$$T_u \propto \sqrt{\frac{1-\delta}{C_{22}}}, \tag{2}$$

without significant dependency on the shape of the core, what confirms the formula already known for a spherical core. So, the longitudinal motion depends on the size of the core, but not on its shape. The influence of the polar flattening ϵ_3 is given in Tab.1. We see in particular that with a small flattening, the period of the free librations of the obliquity T_v tends to ∞, and that the period of the free core nutation T_z gets closer to the orbital period 1.77 d. So, ϵ_3 has actually an influence.

Table 1: Influence of the parameter ϵ_3, with $\delta = 0.5$.

ϵ_3/ϵ_1	T_v (d)	T_w (d)	T_z (d)
0.2	6414.8030	117.1225	1.7275
0.3	2491.6657	117.1166	1.7266
0.5	1163.4490	117.1504	1.7248
1	547.7320	117.5013	1.7198
5	210.4693	142.2052	1.6684
7	194.1446	168.1116	1.6415
9	185.6225	278.9951	1.6156

4 An unexpected behavior

Numerical simulations have shown that for some values of the parameters, especially for high ϵ_3, the equilibrium $J = J_c = 0$ becomes unstable, and two new stable equilibria appear. The Tab.2 gives the consequences of the observable outputs, i.e. the mean obliquity of the mantle $< K_m >$, the mean amplitude of the polar motion of the mantle $< J_m >$, and the one of the fluid $< J_c >$. We can see a significant polar motion of several degrees. An analytical study allows us to give an analytical (and complicated) equation showing the existence or not of these new stable equilibria, for any body in 1:1 spin-orbit resonance for which the Poincaré model can be applied.

Table 2: Observable outputs. The first case is the reference one, and the other ones correspond to the cases where two additional stable equilibria appear.

ϵ_3/ϵ_1	ϵ_4/ϵ_2	$< K_m >$	$< J_m >$	$< J_c >$
1	1	2.30 am	0.15 as	21.47 as
9.45	0	9.21 am	3.70°	3.39°
10	0	37.52 am	14.98°	13.72°
10	0.3	31.21 am	12.56°	11.52°

5 Conclusion

This general study can be applied to any body in 1:1 spin-orbit resonance. For each of them, a specific study of the interior structure must be performed to estimate the possible rotational behaviors, in particular to check whether a small polar motion can be stable or not.

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