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Is Titan in hydrostatic equilibrium?

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Abstract

The observed ratio of the degree-2 gravity coefficients of Titan, $J_2/C_{22} \sim 10/3$, is consistent with a degree-2 gravity field and moments of inertia dominated by hydrostatic rotational and tidal deformation. We consider the effect of non-hydrostatic contributions to validate the significance of this ratio. We illustrate that the observed ratio is also consistent with a degree-2 gravity field and moments of inertia dominated by fossil figure and/or mass anomaly contributions. However, a dominant fossil figure contribution is unlikely since the expected orbital migration is small, and a dominant mass anomaly contribution can be rule out given the observed power spectrum of the gravity field.

1. Introduction

The Radau-Darwin approximation is commonly used to estimate the mean moment of inertia of planetary bodies under the assumption of hydrostatic equilibrium. Iess et al. [1] estimated a mean moment of inertia for Titan $I = 0.34\,mr^2$, where m and r are the satellite mass and radius, using gravity data and the Radau-Darwin approximation.

If Titan is in hydrostatic equilibrium, the degree-2 gravity harmonic coefficients can be written as [2]

$$J_2 = \frac{1}{6} k_2^T \frac{Mr^3}{ma^3} \left[Q_T - 2Q_R \right]$$
$$C_{22} = \frac{1}{12} k_2^T \frac{Mr^3}{ma^3} Q_T, \quad (1)$$

where M is Saturn's mass, a is Titan's semimajor axis, and k_2^T is the secular degree-2 tidal Love number [e.g. 3]. In Eq. (1),

$$Q_R \equiv -p^2 + \frac{3}{2}\left[H(p,\, e) - (1-e^2)^{-3/2} \right]$$
$$Q_T \equiv 3H(p,\, e) \quad (2)$$

are dimensionless coefficients characterizing the distortions around the rotation and tidal axes respectively. These coefficients are functions of the spin-orbit resonance coefficient, p, defined as the ratio between the rotation rate and the mean motion, and the orbital eccentricity, e. In Eq. (2), $H(p,\, e)$ are Hansen coefficients, commonly used in expansions of elliptical motion [e.g., 4, Table 1]. Since Titan is in synchronous rotation, $p = 1$ and $H(p,\, e) = 1 - 5e^2/2 + \cdots$, where we ignore higher order terms in eccentricity.

The ratio

$$\frac{J_2}{C_{22}} = 2\left(1 - 2\frac{Q_R}{Q_T}\right) \quad (3)$$

is a function of the spin-orbit resonance ratio and orbital eccentricity; however, it is independent of the secular degree-2 tidal Love number and the semimajor axis. It is useful to consider the case of $e \ll 1$ since Titan's orbital eccentricity is 3%. In this case; $Q_R \sim -1$, $Q_T \sim 3$, and $J_2/C_{22} \sim 10/3$. The expected ratio of $10/3$ is in agreement with the value estimated using Cassini, Pioneer, and Voyager data, and astronomical observations of Saturn and its satellites [1]. This has been taken as evidence that the degree-2 gravity field and moments of inertia of Titan are dominated by hydrostatic rotational and tidal deformation. We consider the effect of non-hydrostatic contributions to validate the significance of the ratio between J_2 and C_{22}.

2. Fossil figure

If the rotation rate, semimajor axis, or orbital eccentricity of Titan has changed since the time finite rigidity is established, a fossil figure can preserve a record of remnant rotational and tidal deformation. In this case, the gravity coefficients have equilibrium contributions that describe the deformation in response to the present rotational and tidal potentials, and fossil figure contributions that describe the remnant rotational and tidal deformation [2]. We will illustrate that a fossil figure due to changes in rotation rate and/or orbital eccentricity is not consistent with the observed J_2/C_{22}. On the other hand, we will also show that for a fossil figure due to changes in semimajor axis alone, J_2/C_{22} is equal to the expected hydrostatic value of $10/3$. Thus, the observed J_2/C_{22} is also consistent with a gravity field and moments of inertia dominated by a fossil figure due to orbital migration.

Zebker et al. [5] argued that the observed, non-hydrostatic, degree-2 topography of Titan can also be explained by a fossil figure due to orbital migration. However, we will show that a fossil figure due to orbital migration alone cannot explain the observed degree-2 topography. Furthermore, significant orbital migration is unlikely given the expected tidal dissipation within Saturn [6]. If the observed J_2/C_{22} is due to hydrostatic rotational and tidal deformation, the non-hydrostatic topography can be explained by compensated mass anomalies [6].

3. Mass anomalies

Mass anomalies can also contribute to the gravity field and moments of inertia of Titan. Figure 1 shows the probability density of J_2/C_{22} from 10^5 simulations of randomly distributed mass anomalies. The probability density is not sensitive to the number or size of the mass anomalies. The maximum of the probability density corresponds to a J_2/C_{22} value that is similar to the hydrostatic value of $10/3$. Therefore, the observed J_2/C_{22} is also consistent with a gravity field and moments of inertia dominated by mass anomalies.

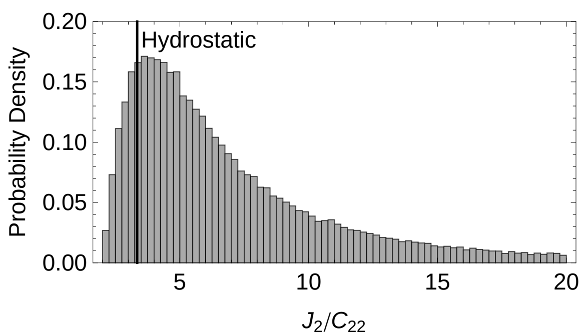

Figure 1: Probability density of J_2/C_{22} from 10^5 simulations of randomly distributed mass anomalies. The vertical line shows the expected value for Titan assuming hydrostatic equilibrium, $J_2/C_{22} \sim 10/3$.

If Titan is in hydrostatic equilibrium, the power spectrum of the gravity field must be dominated by degree 2. Thus, the observed degree-2 and degree-3 gravity coefficients provide an additional constraint [1]. We will illustrate that the observed power spectrum rules out a gravity field dominated by mass anomalies.

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