EPSC Abstracts
Vol. 6, EPSC-DPS2011-345-1, 2011
EPSC-DPS Joint Meeting 2011

Mimas 5:3 Density Wave in Saturn's Rings

N. Rappaport (1), S. Asmar (1), R. French (2), E. Marouf (3) and C. McGhee (2)
(1) JPL/Caltech, CA, USA, (2) Wellesley College, MA, USA, (3) San Jose State University, CA, USA
(Nicole.J.Rappaport@jpl.nasa.gov / Tel: +1-678-603-1692)

Abstract

This paper presents a new, much improved method to analyze in combination many rings occultations profiles using the nonlinear theory.

1. Introduction

Cassini Radio Science ring occultation experiments provided significant radial and azimuthal information on the structure of Saturn's rings, which allows us to determine the kinematics of density wave and to better understand some physical processes operating in rings as well as in extra-solar system .

1.1. Data

We used X-band data acquired during 12 occultations in the period from May to September 2005. The raw data were pre-processed, especially to remove diffraction effects and were provided to the scientists in the form of normal optical depth profiles for each experiment, together with additional information needed for data analysis.

1.2. Motivations

The first motivation to analyze the Mimas 5:3 density wave is that this is the quintessential nonlinear density wave, so it is a very good example to choose for verifying the nonlinear theory of density wave.

Density waves provide information on the rings' physical properties, especially the surface density and to a lesser degree the particles' velocity dispersion. These facts are used by many scientists, mostly to determine the surface density. We have devised a new method to obtain these properties, not just as one or two numbers for a given density wave, but as functions of the semi-major axis.

Our work gives us all the information needed to compute the nonlinear torque from the observations via a single quadrature, so that we can compare our value with the one computed from the linear theory formula. We are also planning to apply the same analysis to a linear, well-behaved, linear density wave and do a similar comparison.

We have many other tasks in mind for the longer range. Just as an example, we intend to study the different modes of dissipation in various ring locations.

1.3. First Procedure

We started this work with P.-Y. Longaretti and devised a first procedure for analyzing the data. Unfortunately, this procedure suffered from inumerable problems; we could neither compute the torque nor investigate the dissipation of waves.

2. Definitions of Variables and Fundamental Equations

The ring particles orbit Saturn on streamlines whose shapes are given by:

$$r = a\left[1 - e(a)\cos\left(m\phi + m\Delta(a)\right)\right], \qquad (1)$$

where r is the radius, a is the semi-major axis, $e(a)$ the eccentricity, $m\phi$ is the resonance angle and $m\Delta(a)$ is a lag angle.

About Eq. (1) it is important to note that we have two variables, the radius r and the semi-major axis a. The approximation $r \simeq a$ must not be made. The inversion of the function $r(a)$ cannot be done analytically because it does not converge. A major source of headaches with the first procedure was the constant change of variable in the functions between a and r and vice-versa.

So what are these functions? We already introduced $r(a)$, $e(a)$, and $m\Delta(a)$. The optical depth is given by:

$$\tau(a) = \frac{\tau_0(a)}{1 - q(a)\cos\left(m\phi + m\Delta(a) + \gamma(a)\right)}, \qquad (2)$$

where $\tau_0(a)$ is the background surface density, and $q(a)$ and $\gamma(a)$ are nonlinearity parameters defined by:

$$q(a)\cos(\gamma(a)) \quad = \quad a\frac{de(a)}{da},$$

$$q(a)\sin(\gamma(a)) \quad = \quad mae(a)\frac{d\Delta(a)}{da}. \qquad (3)$$

We see that we have a lot of intermingled equations.

In addition to these, let us mention the determination for each profile of the resonance radius, the resonant angle ϕ, and the pattern speed. These computations required using the epicyclic theory for the motion of Mimas around the times of the occultations. The calculation of ϕ and its verification to our satisfaction took several months (the values computed in our "first procedure" paper were incorrect).

3. Second Procedure

In this section, we do not describe the procedure per se, but we cite the basic ideas on which it was built.

3.1. Basic Ideas

FIRST BASIC IDEA

This was the realization that the full kinematics of a density wave is actually described by only three functions. We called these functions the "Basic Functions." They are $\tau_0(a)$, $y = ares \times e(a)$, and $z = m\Delta(a)$. All the other functions can be derived from the "Basic Functions" and for this reason we call them the "Derived Functions."

SECOND BASIC IDEA

Both the basic and derived functions are smooth functions of the semi-major axis while they oscillate when they are expressed as functions of the radius. Therefore came the simple idea of using only a semi-major axis model in all the calculations, except for the very last one.

THIRD BASIC IDEA

Represent the basic functions, and by consequence the derived functions, by utilizing various analytical functions which are very well known. Our 19 fit variables were simply parameters of these well-known analytical functions.

FOURTH BASIC IDEA We can solve the kinematics problem in a single step, via a very simple least square fit to the data which are the cumulated observed optical depths as a function of the radius for the twelve profiles. And even better, the Jacobian is computed analytically, which makes the program converge very fast.

The procedure does not use a single assumption or approximation. It strictly implements the equations of section 2.

3.2. Results

We obtained very good fits of the data (see Fig. 1) . The basic functions (see Fig. (2) and derived functions have the behavior expected from physical considerations. However, the wavenumber was found to behave far from linearly as a function of the semi-major axis. One of our discoveries is that the rings start to be perturbed much before the resonance radius.

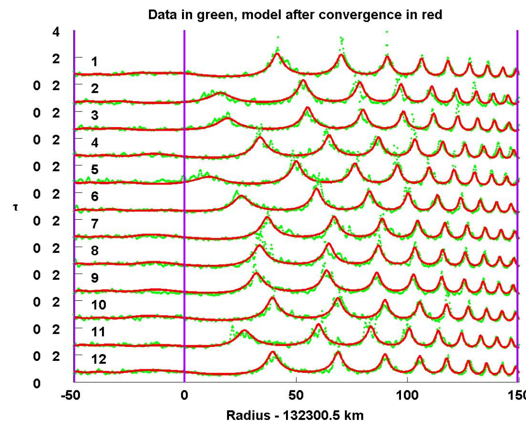

Figure 1: Data and model obtained after convergence.

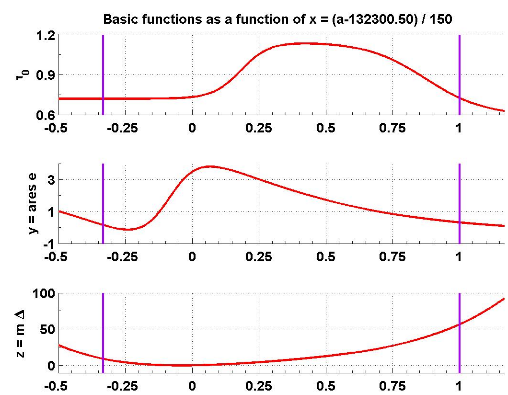

Figure 2: Basic functions after convergence.

Acknowledgements

We are thankful to NASA and the CDAP program, all the people of the Cassini Program and the DSN. Many people provided amazing and unforgettable support that went miles beyond the call of duty. Each author thanks their own Institution for their support.