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Numerical Study of Density Waves in Protoplanetary Disks

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Abstract

With very high spatial resolution, accurate numerical solver, and precise planetary potential, we carry out 2D shearing sheet simulations to provide detailed quantitative comparisons of numerical results with both linear and nonlinear analytical theories on the density waves excited by planets in protoplanetary disks. We achieve very good agreement (at the level of *several percent*) with theories on *primary* physical variables. Also, we study issues which are ignored by the theories, such as the nonlinear effect in the linear stage. The effects of various numerical parameters are extensively investigated, and we provide a framework for future code test in the field of planet-disk interactions. Finally, we discover a commonly ignored but important numerical issue in simulations, which if not handled correctly will lead to incorrect simulation results and an artificial gap opening phenomenon.

1. Introduction

The discovery of "hot Jupiters" residing on very tight orbits from their host stars and the short type I migration timescale compared with the life time of the protoplanetary disks motivate the study of planetary migration, in which the density waves excited by planets embedded in protoplanetary disks play a central role. The interactions between the waves and the embedded planet transfer angular momentum between the disk and the planet, ultimately resulting in the orbital migration of the planet. At the meantime, density waves can drive global evolution of the disk, leading to the migration feedback which slows down the type I migration and potentially gap opening, once they are damped and the carried AMF is transferred to the disk material. Thus, a better understanding of the density wave properties and processes of their excitation, propagation, and damping plays a crucial role in the global picture of planet formation.

The analytical theories of density wave evolution in protoplanetary disks have been firmly established in both the linear [1] and nonlinear regimes [2]. Since then, many numerical works have emerged to check the theories with simulations, but most of them focused on *secondary* (derived) quantities such as the planetary migration rate or the gap opening mass. Here we carry out detailed numerical explorations of density wave excitation and propagation in both linear and nonlinear regimes, and provide quantitative comparisons with analytical results in terms of the spatial distributions of the surface density perturbation, planetary torque (in both physical and *Fourier* space), the growth and damping of the angular momentum flux (AMF) carried by the wave, potential vorticity generation at the shock, and the shocking distance dependence on the planetary mass.

2. Code and Numerical Setup

We carry out 2D shearing box ($x = r - r_{\rm p}$ and $y = r_{\rm p}(\theta - \theta_{\rm p})$) inviscid hydrodynamics simulations using Athena, a grid-based code using higher-order Godunov methods [4]. The simulations are done with 3rd order accuracy Roe Riemann solvers and with spacial resolution 256 grid points per scale height (h). We explore both the isothermal equation of state (EOS) and an adiabatic EOS with $\gamma = 5/3$, and use very low mass planets (a few lunar to a few Earth masses at 1 AU in the MMSN model; the simulation mass unit is $M_{\rm th} = \frac{c_{\rm s}^3}{\Omega_{\rm p} G}$, where $c_{\rm s}$ is the sound speed) to better separate the linear and nonlinear regimes. The planetary potential is implemented as

$$\Phi_{\rm p} = -{\rm G}M_{\rm p}\frac{\rho^2 + 1.5r_{\rm s}^2}{(\rho^2 + r_{\rm s}^2)^{3/2}},\ (\rho = \sqrt{x^2 + y^2}) \quad (1)$$

which converges to the Kaplerian potential $(-{\rm G}M_{\rm p}/\rho)$ as $(r_{\rm s}/\rho)^4$ for $\rho \gg r_{\rm s}$. The numerical viscosity is below Shakura-Sunyaev $\alpha = 10^{-5}$ for typical runs.

3. Results

Our results show quantitatively good agreement with theory in both *physical* and *Fourier* space, as Fig. 1 showing two examples. Specifically, in panel (a) the evolution of the numerical torque at the small m end with x is because the wave collects smaller and smaller $m_{\rm th}$ resonance on its way away from the planet, while the growth of power at the high m end is because the nonlinear effect becomes prominent at large distance even when the wave is formally linear. Other manifestations of the nonlinearity in the linear stage include the steepening of the density profile, and the slight depression of the asymptotic torque value at large planetary mass (in addition to the expected $\propto M_{\rm p}^2$ dependence), causing a deviation from analytical results by several percent. In the linear regime, we persistently find a disagreement with the theory in the form of a change of sign in the torque density calculation at a fixed distance from the planet, though it only weakly affects the total torque on the planet. In the nonlinear regime, we study the potential vorticity generation at the nonlinear shock, and use it to pin point the shock location. In addition, we find that the linear damping due to viscosity, even at the level of $\alpha = 10^{-4}$, can strongly affects shock formation, the nonlinear wave evolution, and the AMF damping process.

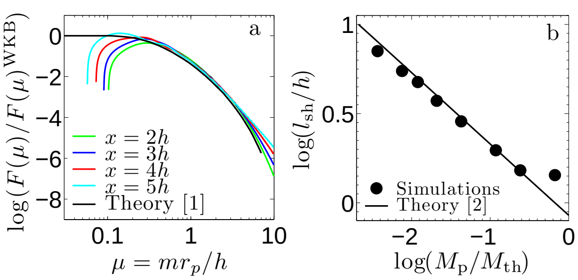

Figure 1: Comparisons with the analytical theories on (a) the ratio of the torque at each $m_{\rm th}$ Lindblad resonance to its value under WKB approximation (*i.e.* the torque cut off in Fourier space), and (b) the shocking length ($l_{\rm sh}$, the orbital distance between the planet and the shock in the wave resulting from the nonlinear evolution) dependence on $M_{\rm p}$. The black line in each panel indicates the analytical result in [1] and [2].

We investigate the effects of various numerical parameters, and find that high spatial resolution, highly accurate numerical solver, and precise planetary potential are crucial for achieving excellent agreement with the analytical theories. Specifically, we contrast the results of our fiducial simulation (spatial resolution $256/h$, 3rd order accuracy solver, and potential (1) with softening length $r_{\rm s} = h/32$) with another one employing typical numerical parameters in recent literature (spatial resolution $32/h$, 2nd order accuracy solver, and potential $\Phi_{\rm p} = -{\rm G}M_{\rm p}/(\rho^2 + r_{\rm s}^2)^{1/2}$ with $r_{\rm s} = h/4$). Comparing with theory, our fiducial simulation demonstrates good agreement, while the other one produces a density perturbation at lower amplitude (by $\sim 20\%$), a lower asymptotic final torque (by $\sim 30\%$), and most importantly, an largely advanced nonlinear wave damping (a factor of 2 in distance), which further affects the global disk evolution, such as the migration feedback and the gap opening. We note that the way in which we make the comparison with the theory and the agreement we achieve may serve as a standard framework for future code tests.

We discover a generally ignored numerical issue in disk-planet simulations. To resolve the motion of the fluid in regions where the fluid experiences the largest gravitational acceleration, the numerical time step dt should be larger (by a certain factor) than the limiting (smallest) dynamical timescale in the simulation domain (which in disk-planet simulations usually occurs at the grid points closest to the planet for normal planetary potential implementations). We find when dt violates this condition (while still satisfying the Courant condition), the simulation produces an numerical artifact which is visually very similar to (and can easily be confused with) the gap opening phenomenon. This issue applies to all simulations with point masses in disks, and particularly affects the low resolution ones ($32/h$ or lower), and the ones using the Fast Advection in Rotating Gaseous Objects technique (FARGO [3], which results in a much larger time step dt than usual). In our simulations with FARGO, runs with $M_{\rm p} \sim M_{\rm th}$ and a large $r_{\rm s} = h/4$ must have an effective resolution at least $64/h$ in the vicinity of the planet to avoid this problem (a more smoothed potential may weaken this condition, through).

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