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NAFFO: A numerical algorithm to simulate the damped rotation of synchronous satellites

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Abstract

We hereby introduce and study an algorithm able to determinate the initial conditions corresponding to the equilibrium rotational motion of tidally evolved natural satellites locked in spin-orbit resonance, hereby named Namur Algorithm For Forced Oscillations, NAFFO for short.

NAFFO is based on the Numerical Analysis of the Fundamental Frequencies algorithm by J. Laskar, for the identification of the free and forced oscillations, the former being iteratively removed from the solution by carefully choosing the initial conditions.

We proved the convergence of the algorithm under suitable assumptions, satisfied in the Hamiltonian framework whenever the d'Alembert characteristic holds true. In that case, the convergence is quadratic.

1 Introduction

The main natural satellites of the Solar System are assumed to be tidally evolved and to have reached the 1:1 spin-orbit resonance, i.e. the synchronous rotation. This is a dynamical equilibrium state around which free librations are expected to be sufficiently damped to be negligible. If we want to simulate this rotational state in a simplified system, i.e. in assuming a circular or a keplerian orbit around their parent planet, this equilibrium can be accurately derived. However, if we want to consider a complete motion of the satellite around its planet, oscillations due to real orbital motion of the satellite impede an accurate determination of this equilibrium. As a consequence, a numerical simulation of the rotation would result in artificial free librations with non negligible amplitudes.

We here present a numerical algorithm, named NAFFO (Namur Algorithm For Free Oscillations), that iteratively converges to the exact equilibrium. This algorithm is based on Laskar's frequency analysis NAFF [2]. After a presentation of this algorithm, we show a concrete application on a problem of rotational dynamics.

2 The algorithm

As an example, let us consider a simplified planet-satellite system in which the orbital motion of the considered satellite is ruled by the proper sinusoidal mode λ (in a multi-dimensional system, there would be several modes λ_i). Its rotational dynamics involves an additional free mode u representing free librations around the equilibrium. This last mode is considered to be damped and we want to numerically get the solution of the problem corresponding to this state. In calling $x(t)$ the solution of the problem, a numerical integration of the equations of motion followed by a numerical determination of the proper modes will give the following quasi-periodic decompositions, in the Hamiltonian framework:

$$\begin{aligned} x(t) &= \sum_{k,p\in\mathbb{Z}} \alpha_{k,p}\sqrt{U}^{|k|} \\ &\times \left(1+\sum_{m\geq 1}\gamma_{m,k,p}U^m\right) \\ &\times \exp\Big(\imath\big(ku(t)+p\lambda(t)\big)\Big), \end{aligned} \tag{1}$$

where U is the amplitude associated with the free librations. The powers in the amplitudes are given by d'Alembert rule.

We want to get the solutions corresponding to $U = 0$, for that we follow the following algorithm:

1. Choose initial conditions sufficiently close to the equilibrium (for instance, analytically determined for a simplified model)

2. Numerically integrate the equations of the rotational motion

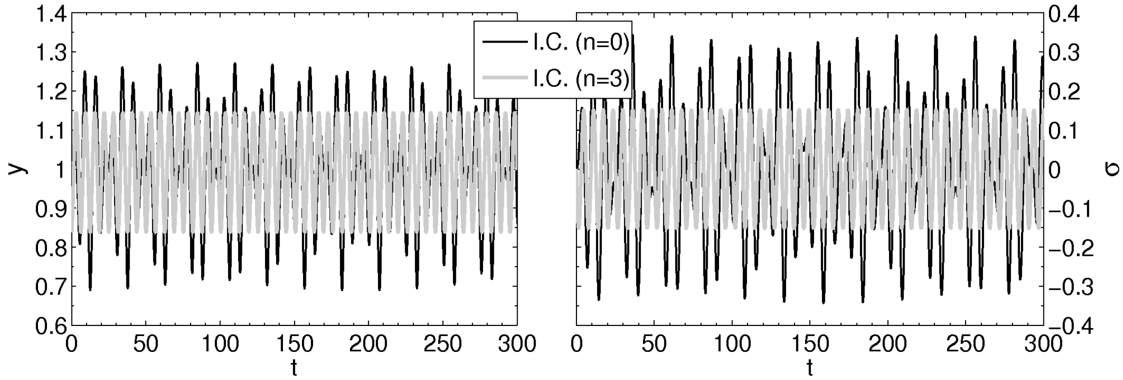

Figure 1: Application of NAFFO in a difficult case. The grey curves represent the forced solution, whose initial conditions have been obtained after 3 iterations. The initial solution, i.e. the black curves, contains free oscillations.

3. Perform a frequency analysis of the solutions
4. The free part of the solutions, considered as the one containing the proper mode u, is numerically evaluated at the initial date and removed from the initial conditions. These new initial conditions are then used in step 2.

We have shown in [4] that this algorithm has a quadratic convergence in the Hamiltonian framework. It can also be used in a more general context under suitable hypotheses on the amplitudes associated with the free modes, that are usually verified.

3 Application

The Hamiltonian 1-dimensional spin-orbit motion can be represented by the resonant argument $\sigma = \lambda - x$ where λ is the mean longitude of the satellite in spin-orbit resonance, and x the orientation of its long axis, and its associated momentum y. It represents the norm of the angular momentum of the body, that should be close to 1 in this problem. The dynamics of this simplified system also depends on 2 parameters: the eccentricity e, and the equatorial ellipticity of the satellite ϵ.

We present the behavior of NAFFO for a difficult case with $\epsilon = 0.3$ and $e = 5.49 \times 10^{-2}$, i.e. when the free frequency ω gets close to the orbital frequency, what complicates the determination of the proper modes. The Fig.1 shows the solutions before ($n = 0$) and after 3 iterations of NAFFO. The resulting orbit does not contain the free oscillations of the resulting orbit anymore. In a more realistic case like the Earth-Moon system, the convergence is more efficient.

4 Conclusion

We have here presented NAFFO in a simplified case, but we have used it successfully in 4 d.o.f. systems for studying the spin-orbit resonant dynamics of Mercury [3], including polar motion and pressure coupling at the core-mantle boundary. This algorithm can also be used in a more general context like ground-track resonances [1].

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