Effects of asteroids on the orbital motions of terrestrial planets

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Abstract

This work is presented in order to try to assess with as much detail as possible the effects of the largest asteroids on the orbits of terrestrial planets Mercury, Venus, Earth and Mars. The present planetary ephemerides, as INPOP08 (Fienga et al 2009) and DE405 (Standish et al. 1998) are subject to a lack of accuracy, because of the perturbations arising from a large number of asteroids. Those perturbations could reach a few kilometers in several decades in the case of Mars for instance. So it looks appropriate to model in details the individual specific effects of these asteroids. For that our methodology consisted of several stages: a numerical integration of the orbits of the planets with and without the disturbing asteroid from which we want to know the effects; a determination of the signal representing the effects, by simple subtraction; an analysis of this signal by FFT (Fast Fourier Transform / Fast Fourier Transform); and adjustment of the signal by the set of sinusoids determined in the previous step.

This type of study is interesting in many fields, such as planetary ephemerides, as well as spatial navigation, to understand better the effects of each asteroid taken individually on the terrestrial planets. Note that this type of study is a continuation of previous studies (Williams 1984; Mouret et al 2009).

1. Introduction

The motion of a given planet around the Sun can be considered at first approximation as a Keplerian motion perturbed by the other planets and the small bodies of the solar system. Each of these perturbations must be treated either analytically or numerically, and can be measured as a change of the planet’s osculating orbital elements \( a, e, i, \Omega, \varpi = \Omega + w \) and \( L = \omega + M \) determined from the perturbing function \( \mathcal{R} \), according to Lagrange’s formula:

\[
\begin{align*}
\frac{da}{dt} &= \frac{2}{na} \frac{\partial \mathcal{R}}{\partial \mathcal{L}} \\
\frac{de}{dt} &= -\frac{\sqrt{1-e^2}}{na^2 e} \left( 1 - \sqrt{1-e^2} \right) \frac{\partial \mathcal{R}}{\partial \mathcal{L}} - \frac{\sqrt{1-e^2}}{na^2 e} \frac{\partial \mathcal{R}}{\partial \varpi} \\
\frac{di}{dt} &= -\frac{1}{na^2 \sqrt{1-e^2} \sin i} \left[ \frac{\partial \mathcal{R}}{\partial \Omega} + (1 - \cos i) \left( \frac{\partial \mathcal{R}}{\partial \varpi} + \frac{\partial \mathcal{R}}{\partial \mathcal{L}} \right) \right] \\
\frac{d\Omega}{dt} &= \frac{1}{na^2 \sqrt{1-e^2} \sin i} \frac{\partial \mathcal{R}}{\partial \Omega} \\
\frac{d\varpi}{dt} &= \frac{\sqrt{1-e^2} \frac{\partial \mathcal{R}}{\partial \mathcal{L}}}{na^2 e} + \frac{1 - \cos i}{na^2 \sqrt{1-e^2} \sin i} \frac{\partial \mathcal{R}}{\partial \varpi} \\
\frac{dL}{dt} &= \frac{n}{na} \frac{\partial \mathcal{R}}{\partial \mathcal{L}} + \frac{\sqrt{1-e^2}}{na^2 e} \left( 1 - \sqrt{1-e^2} \right) \frac{\partial \mathcal{R}}{\partial e} + \frac{1 - \cos i}{na^2 \sqrt{1-e^2} \sin i} \frac{\partial \mathcal{R}}{\partial \varpi}
\end{align*}
\]

2. Effects on Mars

To evaluate the effects of a given asteroid on the terrestrial planets we use the numerical integration (Runge-Kutta of the 12th order), in the frame of the 9-body problem (the Sun and the eight planets without asteroids), then of the 10-body problem (the Sun and the eight planets together with the given asteroid). Then we determine the differential variations of orbital parameters of the planet by simple subtraction of the two signals obtained. After we perform the frequency analysis of the data, using fast Fourier transform (FFT) to determine the leading frequencies. At last we carry out a nonlinear regression in which the differential data are modeled by least-square method following an equation of type:

\[
F(t) = \sum_{i=1}^{N} A_i \sin(f_it) + B_i \cos(f_it) + C_i t \sin(f_it) + D_i t \cos(f_it)
\]

We present below an example of our results: in the next Table we show the maximum effects of each asteroid on the orbital parameters of Mars, for the periods of 100 years.
The corresponding curves is presented in the next Figure (the initial signal in black, the adjustment determined by our analysis in red and the residuals in green). In each case: (planet, asteroid, orbital element) we find that our fit is satisfactory.

The following Table shows the coefficients of Fourier and Poisson components for the effects of Ceres on the semi-major axis of Mars.

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<th>Asteroid</th>
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<th>10^-6</th>
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<tr>
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<td>837.97915</td>
<td>37190.92552</td>
<td>263.123433</td>
<td>16391.84563</td>
<td>19419.20264</td>
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<td>16.94125</td>
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<tr>
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<td>0.12028</td>
<td>4.48961</td>
<td>2.69194</td>
<td>8.82521</td>
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<tr>
<td>Emma</td>
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<td>1.04708</td>
<td>16.94125</td>
<td>9.49715</td>
<td>9.82103</td>
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The next Figure presents the amplitude with respect to each frequency found from our analyses of the individual influence of asteroids on the distance EMB-Mars. There are very clear vertical lines, related to the synodic period of the Mars.

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References