

# Viscosity in Planetary Rings: Results from Numerical Simulations

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## Abstract

Saturn's rings are composed of many icy particles, and angular momentum is transported due to collision and gravitational interaction between these particles. Viscosity in the rings arising from such interactions between particles governs the rate of dynamical evolution and structure formation in the rings. We examine the effect of surface friction on the viscosity and the dependence of the viscosity on optical depth and distance from Saturn, using local N-body simulation. We derive semi-analytic formula that approximately reproduces our numerical results.

## 1. Introduction

Viscosity in Saturn's rings has been investigated with various approaches. In earlier theoretical studies the viscosity was estimated neglecting mutual gravitational forces between particles [1,2]. However, local N-body simulations including collision and mutual gravitational forces between particles [3-6] showed that, in optically thick rings, wake structures are formed due to their self-gravity. Observations by the Cassini spacecraft as well as various ground-based observations show results consistent with the existence of wake structures.

Viscosity in such self-gravitating dense rings has been studied using local N-body simulations that take account of collision and mutual gravitational forces between particles [7]. Numerical results show that the viscosity is significantly enhanced due to the effect of self-gravity in dense rings with gravitational wake structures. The above N-body simulations assumed smooth spherical particles and did not examine effects of surface friction and particle's spin.

Spins of ring particles are produced as a natural outcome of oblique impacts between particles with surface friction. The spin rate of ring particles is one of the important parameters in the modeling of

thermal emission from Saturn's rings, and has been studied in detail, using analytic calculation, three-body orbital integration, and N-body simulation [8-12]. These studies show that the spin period is on the order of the particles' orbital period in rings of equal-sized particles, while small particles spin faster than larger ones when size distribution is included. Considering energy balance between viscous gain and dissipation due to inelastic collision with surface friction, viscosity in planetary rings consisting of spinning, non-self-gravitating particles has been estimated [12], but effects of particles' surface friction and spins on the viscosity in self-gravitating rings has not been studied in detail. In the present study, we examine the viscosity in planetary rings, taking effects of particle spins into account, in addition to collision and gravitational interaction between particles.

## 2. Method

Assuming that a local region in a ring is in a quasi-steady state, the viscosity in such a region can be estimated via the balance between energy loss due to inelastic collisions and the viscous gain due to the shear motion, even in the case of self-gravitating rings [13]. A similar method for viscosity calculation was applied to non-self-gravitating rings in relation to the study of the viscous overstability [14]. It has been shown that viscosities in self-gravitating rings can be evaluated with this method [13] and that results agree with those obtained in a different method [7].

We adopt the method of local N-body simulation, which was used in the previous studies [e.g., 2-7]. We use a code developed and used by [5] and [7]. We consider a computational box which is at an arbitrary semi-major axis from Saturn and revolves along a circular orbit with the Keplerian angular velocity around Saturn. We set the width of the box

to be much smaller than scale of Saturn's rings but sufficiently larger than the typical scale of wakes. To calculate the orbits of particles, we solve the equation of motion under Hill's approximation numerically. We suppose eight copied boxes surrounding the computational box. Periodic boundary conditions for the computational box are adopted, taking account of the shear motion of the above copied boxes. In calculating mutual gravitational forces between particles, the computational box is subdivided into nine subregions and we make a virtual area for each subregion in which the subregion is centered and the virtual area has the same size as the original computational box. We calculate directly gravitational forces exerted on particles in the subregion from particles in the virtual area using GRAPE-7, which is a special-purpose hardware for calculating gravitational forces. The orbits of particles are integrated with the second-order leap frog method. When a collision between particles is detected, we calculate the change of velocity due to collision using given restitution coefficients in the normal and tangential directions. In the case with surface friction, we also calculate the change of particle's spin rates.

We investigate the dependence of the viscosity on various parameters such as optical depth, distance from Saturn, and normal and tangential restitution coefficients. We also calculate the viscosity with the velocity-dependent restitution coefficient based on laboratory impact experiments [15]. Furthermore, we obtain semi-analytic expression that approximately reproduces our numerical results.

### 3. Results

First, we calculate viscosities in the case without surface friction. In the case of low optical depth, the viscosity was found to increase in proportion to the optical depth, and excellent agreement with the results based on three-body calculation was confirmed. However, in dense rings where gravitational wakes are formed, the results and the viscosity is significantly enhanced, in agreement with the previous results for rings with self-gravitating, smooth particles [7]. Next, we examine the effect of surface friction and spins of particles. In the case of rings with low optical depth and weak self-gravity, both particle random velocities and viscosities are slightly lowered due to surface friction. On the other hand, in the case of optically thick rings in which wake structures are strongly formed, we found that

the dependence of the viscosity on the tangential restitution coefficient is negligible. This is because, in dense rings, the enhancement of the viscosity due to rings' self-gravity is much more significant than the effect of particles' surface friction.

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### References

- [1] Goldreich, P., and S. Tremaine 1978, *Icarus* 34, 227-239. [2] Wisdom, J., and S. Tremaine 1988, *Astron. J.* 95, 935-940. [3] Salo, H. 1995, *Icarus* 117, 287-312. [4] Richardson, D. C. 1994, *Mon. Not. R. Astron. Soc.* 269, 493-511. [5] Daisaka, H. and S. Ida 1999, *Earth Planets Space*, 51, 1195-1213. [6] Ohtsuki, K. and H. Emori 2000, *Astron. J.* 119, 403-416. [7] Daisaka, H., H. Tanaka, and S. Ida 2001, *Icarus* 154, 296-312. [8] Ohtsuki, K. 2005, *Astrophys. J.* 626, L61-L64. [9] Ohtsuki, K. and D. Toyama 2005, *Astron. J.* 130, 1302-1310. [10] Ohtsuki, K. 2006a, *Icarus*, 183, 373-383. [11] Ohtsuki, K. 2006b, 183, 384-395. [12] Morishima, R. and H. Salo 2006, *Icarus* 181, 272-291. [13] Tanaka, H., K. Ohtsuki, and H. Daisaka 2003, *Icarus* 161, 144-156. [14] Salo, H., J. Schmidt, and F. Spahn 2001, *Icarus* 153, 295-315. [15] Bridges, F., A. P. Hatzes, and D. N. C. Lin 1984, *Nature* 309, 333-335.