



Constraining the Internal Structures of Jupiter and Saturn from Moment of Inertia Measurements: Implications for the Juno and Cassini Solstice Missions

R. Helled (1,2), J. D. Anderson (3), G. Schubert (1), and D. J. Stevenson (4)

(1) UCLA, Los Angeles, CA, USA (2) Tel Aviv University, Tel Aviv, Israel (3) Jet Propulsion Laboratory, Pasadena, CA (4) California Institute of Technology, Pasadena, CA.

(rhelled@ucla.edu / Fax: +1-310-206-3051)

Abstract

The moment of inertia of a giant planet reveals important information about the planet's internal density structure and this information is not identical to that contained in the gravitational moments. The forthcoming Juno mission to Jupiter and the Solstice Mission (Cassini XMM) to Saturn might determine the angular momentum of the planets, and therefore their moments of inertia $\text{NMoI} = C/MR^2$ by measuring the planets' pole precession, and the Lense-Thirring acceleration of the spacecraft (C is the axial moment of inertia, and M and R are the planet's mass and mean radius, respectively). The possible range of NMoI values for Jupiter and Saturn based on their measured gravitational fields using a simple core/envelope model are presented. The model suggests that Jupiter's NMoI lies in the range 0.2629 - 0.2645. Saturn's NMoI is found to be ~ 0.218 . Constraining Saturn's NMoI value, however, is possible only if an accurate determination of Saturn's rotation period is available.

1. Introduction

Despite decades of investigation the internal structures of Jupiter and Saturn, in particular, their core masses are not well determined. The Juno orbiter will measure Jupiter's gravitational coefficients to high accuracy and provide tighter constraints on the high-order gravitational harmonics. Juno's radio science system will provide an accurate determination of the Doppler shift during the 31 gravity-science orbits. Various effects influence the Doppler signal; two of them involve Jupiter's angular momentum and can therefore constrain its NMoI. The first is Jupiter's pole precession and the second is the Lense-Thirring gravitomagnetism relativistic effect. Including these effects in the Doppler analysis can therefore provide a mea-

surement of Jupiter's NMoI. The Solstice Mission will have 42 close orbits that would enable a determination of Saturn's gravitational harmonics up to degree 10. Since the final orbits of the Solstice mission are essentially similar in nature to the orbits of Juno around Jupiter, it is certainly feasible that Saturn's angular momentum could be measured as well.

The NMoI values for planets are often estimated using the Radau-Darwin formula that relates the planetary NMoI and J_2 , the second degree gravitational coefficient. However, the Radau-Darwin relation is only a first order approximation. In fact, there is a range of NMoI values that can fit a given J_2 . Below we use simple core/envelope models of Jupiter and Saturn constrained by their gravity fields (J_2, J_4, J_6) to explore the range of possible NMoI values.

2. NMoI Variation from A Simple Interior Model

We use a model for the planetary density profile which consists of a constant density core and an envelope which is represented by a sixth order polynomial. For a given core density and radius, the polynomial coefficients of the envelope are found by iterating until the gravitational harmonics of the interior models converge to the measured ones. The density profiles fit the measured J_2 and J_4 of Jupiter and Saturn *exactly* and fit their measured J_6 to within the error bars. The NMoI is then computed using the derived density distribution. This approach allows us to test the variation of the NMoI value for a given gravitational field. In order to exclude unphysical density distributions we include the following constraints: (1) The density profile must be monotonic. (2) The core density $< 30 \text{ g cm}^{-3}$. (3) The core mass $< 40 \oplus$ and 30 M_\oplus for Jupiter and Saturn, respectively.

3. Results

3.1. Jupiter

Fig. 1 shows the NMoi values for Jupiter. The range of allowed models is shown in gray. The top of the figure shows the core mass in M_{\oplus} vs. NMoi. The bottom figure shows the range of NMoi vs. core radius. Not many models with small core radius can be found since the core density must stay lower than the maximum core density and yet be larger (or equal) than the envelope's density at the core-envelope boundary (CEB). As the core radius increases more models can be found. However, in order to keep the core mass smaller than $40 M_{\oplus}$ and at the same time keep the density profile monotonic, the number of models that can be found with increasing core radius decreases as the core radius exceeds $\sim 0.2R$. The NMoi value for Jupiter varies from 0.2629 to 0.2645, a 0.6% variation. The ability to constrain Jupiter's core properties would depend on the actual measured NMoi value.

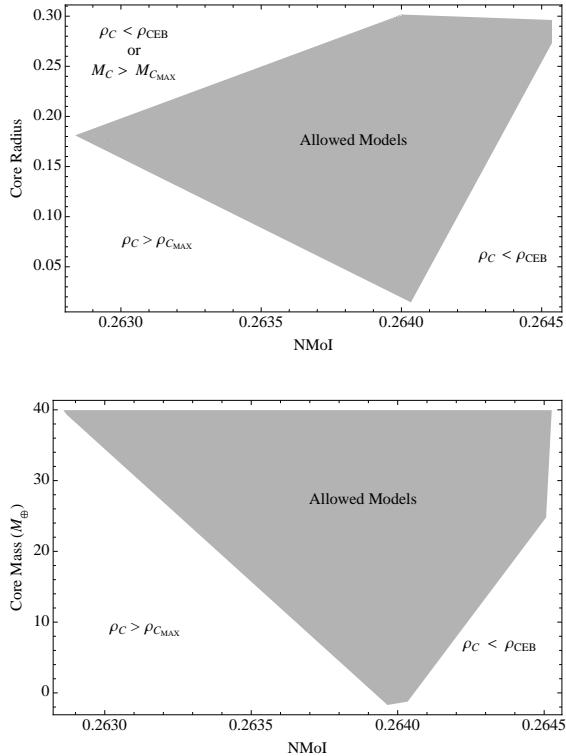


Figure 1: Top: Core radius (normalized to R) vs. NMoi. Bottom: Jupiter's core mass (M_{\oplus}) vs. NMoi value. All the allowed models fit to Jupiter's gravitational field, mass, and shape, and in addition have core properties within the allowed bounds.

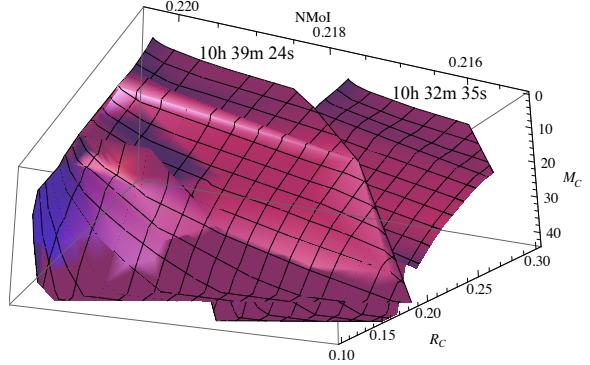


Figure 2: Saturn's allowed models for solid-body rotation periods 10h 39m 24s (Voyager) and 10h 32m 35s (Anderson & Schubert, 2007). The allowed models fit Saturn's gravitational field, mass, and shape, and have core properties within the allowed bounds.

3.2. Saturn

Since Saturn's rotation period is unconstrained within a few minutes, we use two solid-body rotation periods: Voyager's rotation period of 10h 39m 24s, and 10h 32m 35s (Anderson & Schubert, 2007, Sci., 317, 1384). The interior model results are shown in Fig. 2. The range of NMoi values for a given rotation period varies by up to $\sim 2\%$. Although a given NMoi value cannot perfectly constrain Saturn's core properties, the results suggest that an accurate determination of Saturn's NMoi could still provide valuable constraints on its core properties. However, the quantity that is actually measured by the spacecraft is the planet's spin angular momentum $L = C\omega = \text{NMoi}MR^2\omega$ where ω is the rotation rate. The value of Saturn's NMoi cannot be measured separately from L . Saturn's rotation period is not well constrained within a few minutes, and this uncertainty introduces an uncertainty in the NMoi value.

4. Summary and Conclusions

The moments of inertia of Jupiter and Saturn contain important information on their internal structures. Jupiter's NMoi can be measured by the Juno orbiter, while Saturn's NMoi could be determined during the last phase of the Solstice Mission. A determination of the NMoi values of Jupiter and Saturn can provide tighter constraints on the planets' internal structures independent of their gravitational field measurements. Such a determination could provide valuable information on the planets' origin and evolution.