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Quasi-Static Evolution of Self-gravitating Aggregates

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Abstract

Analytical and numerical approaches to understanding the evolution of rubble pile asteroids under increasing angular momentum (due to YORP) are studied. Exact analytical results can be found for few particle systems and provide stringent tests for numerical simulations for rubble piles with large numbers of self-gravitating grains. We test the limits of acceleration rate needed to reproduce quasi-static behavior and how aggregates will change with increasing numbers of grains.

1. Introduction

Rubble-pile asteroids undergoing continued acceleration due to the YORP effect mimic the quasi-static limit often assumed in theoretical continuum mechanics [1], however these results are not directly applicable to discrete particle systems. Since rubble piles are composed of discrete grains, even in the quasi-static limit they are expected to undergo discrete, macroscopic changes in their configuration as their total angular momentum changes. Such reconfigurations for a discrete collection of grains are "plastic," resulting in a deformation of the granular pile into a different configuration. Due to this, rubble piles that reshape under an increasing angular momentum also exhibit hysteresis, meaning that under a subsequent decrease in angular momentum they will not transition at the same angular momentum magnitudes and in the same ways. We present recent advances, both theoretical and numerical, in our modeling of quasi-static processes at rubble pile asteroids.

2. Theoretical Results

Reconfigurations of rubble piles consisting of small numbers of particles can be analyzed and understood completely. Any collection of grains will preferentially assemble itself into a minimum energy configuration, as at this point all relative motion within the rubble pile has dissipated and is also robust against small perturbations that leave the angular momentum unchanged or minimally altered. It can be proven that a minimum energy configuration of a collection of self-gravitating grains will minimize the function $E_{min} = H^2/2I + U$, where H is the total angular momentum of the system, I is the total moment of inertia and U is the gravitational potential energy [2, 3]. Using this functional all of the minimum energy configurations of a collection of self-gravitating grains can be denumerated. Exhaustive minimum energy configurations for $N = 3, 4$ are shown in Fig. 1.

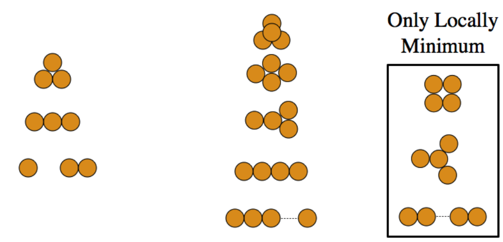

Figure 1: Global and local minimum energy configurations for $N = 3, 4$ for different angular momenta.

A different issue is the identification of angular momentum values at which a rubble pile will reconfigure itself. This occurs when a previously stable minimum energy configuration becomes unstable, and cannot remain in the same configuration but starts to generate relative motion. The computation of these reconfigurations is more difficult, requiring explicit consideration of the relative forces and accelerations acting on the collection of particles. Taking both into consideration, it is possible to delineate the configurations of a rubble pile under increasing and decreasing angular momentum. Fig. 2 shows a reconfiguration chart for $N = 3$.

For systems with few particles, the movement of a single grain can represent a significant change in the configuration. As the number of grains increases, the number of possible configurations that are minimum energy, but not globally minimum, expands factorially, making numerical computation the prime approach to explore these systems.

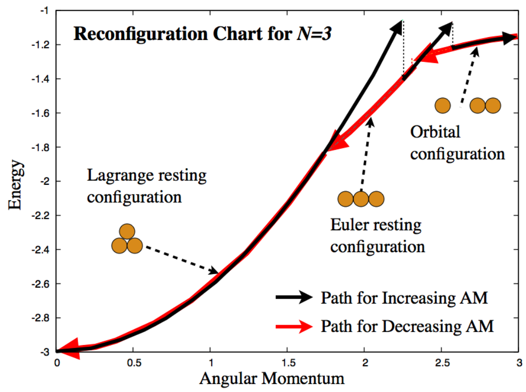

Figure 2: Transition pathways for an $N = 3$ system as angular momentum is increased and decreased.

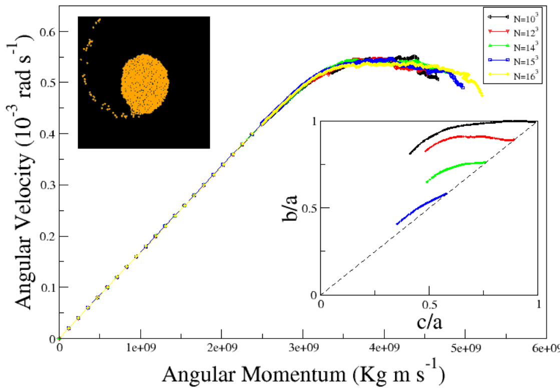

Figure 3: Variation of angular velocity with angular momentum in the quasi-static limit. The inset shows the evolution of the principal axes ratios for different initial shape aggregates.

3. Simulation of N-Body Systems

We have used Soft-Sphere DEM simulations [4] to study the dynamics of a rubble-pile asteroids subjected to quasi-static increase in angular momentum. The model was initially validated against the exact results for few particle systems. Figure 3 shows the angular velocity as a function of angular momentum for different collections of aggregates. Each aggregate is formed by a collection of particles of the same density, but the number and size of these particles is varied so as to keep the total mass of the aggregate constant. The curves show that for a sufficiently large number of particles, the macroscopic behavior of the object is independent the granular resolution.

This limiting behavior suggests that some macroscopic properties of self-gravitating aggregates can be accurately captured using DEM simulations. The inset to Fig. 3 shows the evolution (in the quasi-static limit) of the ratios of the semi-major axes of the DEEVE's, each starting from a different initial shape, and is similar to theoretical curves reported in [1].

For sufficiently high angular velocities the aggregate typically deforms into a pear-like shape and begins to shed material from its apex [5]. At higher values we hypothesize that the aggregate can undergo fission and split into two components.

4. Summary and Conclusions

Granular Mechanics in the Quasi-Static limit have markedly different behavior than in systems that are not allowed to come to rest between each angular momentum increment. Exact results can be found for small systems and used to validate DEM simulations. This work probes the necessary simulation details to capture this phenomenon and investigates how it scales with the number of grains. The simulations also capture the limiting macroscopic behaviour of these aggregates provided that a sufficient number of particles are simulated and a sufficiently slow spin-up rate is achieved.

Acknowledgements

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