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# The sweeping $\nu_6$ secular resonance during giant planet migration: implications for models of the primordial excitation and depletion of the asteroid belt.

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## Abstract

We calculate the eccentricity excitation of asteroids produced by the sweeping $\nu_6$ secular resonance during the epoch of planetesimal-driven giant planet migration in the early history of the solar system. We derive analytical expressions for the magnitude of the eccentricity change and its dependence on the sweep rate and on planetary parameters; the $\nu_6$ sweeping leads to either an increase or a decrease of eccentricity depending on an asteroid's initial orbit. Examination of the orbital data of main belt asteroids reveals that the distribution in proper eccentricities of the known bright ($H \leq 10.8$) asteroids yields two possible solutions for the migration rate of Saturn and for the dynamical states of the pre-migration asteroid belt.

## 1. Introduction

There is abundant evidence that the giant planets of our solar system formed in different orbits than we find them today and later migrated to their present locations [1–9]. As the giant planets migrated, locations of mean motion and secular resonances would have swept across the asteroid belt, raising the eccentricities of asteroids to planet-crossing values, and depleting them from the main belt. We develop here an analytical model for the effect of the sweeping $\nu_6$ secular resonance on the eccentricity distribution of main belt asteroids. The pre-sweeping eccentricity distribution is diagnostic of an event known as the "primordial excitation and depletion" of the main asteroid belt.

## 2. Analytical theory of the sweeping $\nu_6$ secular resonance

We adopt a simplified model in which a test particle (asteroid) is perturbed only by a single resonance, the $\nu_6$ resonance. An asteroid's secular perturbations close to a secular resonance can be described by the following Hamiltonian function [10]:

$$H_{sec} = -g_0 J + \varepsilon\sqrt{2J}\cos(w_p - \varpi), \qquad (1)$$

where $w_p = g_p t + \beta_p$ describes the phase of the p-th eigenmode of the linearized eccentricity-pericenter secular theory for the Solar system planets [11], $g_p$ is the associated eigenfrequency, $\varpi$ is the asteroid's longitude of perihelion, $J = \sqrt{a}\left(1 - \sqrt{1-e^2}\right)$ is the canonical generalized momentum which is related to the asteroid's orbital semimajor axis $a$ and eccentricity $e$; $-\varpi$ and $J$ are the canonically conjugate pair of variables in this 1-degree-of-freedom Hamiltonian system. We will adopt a simple two-planet model of the Sun-Jupiter-Saturn.

During the epoch of giant planet migration, the planets' semimajor axes change secularly with time, so that $g_0$, $g_\mathrm{p}$ and $\varepsilon$ become time-dependent parameters. We approximate a linear change of frequency: $\dot{g}_\mathrm{p} = 2\lambda$. We define $t = 0$ as the epoch of exact resonance crossing [12]. The final value of $J$ long after resonance passage is found to be:

$$J_f = J_i + \frac{\pi\varepsilon^2}{2|\lambda|} + \varepsilon\sqrt{\frac{2\pi J_i}{|\lambda|}}\cos\varpi_i. \qquad (2)$$

The asteroid's semimajor axis $a$ is unchanged by the secular perturbations; thus, the changes in $J$ reflect changes in the asteroid's eccentricity $e$. For asteroids with non-zero initial eccentricity, the phase dependence in equation (2) means that secular resonance sweeping can potentially both excite and damp orbital eccentricities.

For small $e$, we can use the approximation $J \simeq \frac{1}{2}\sqrt{a}e^2$. Considering all possible values of $\cos\varpi_i \in \{-1, +1\}$, an asteroid with initial eccentricity $e_i$ that is swept by the $\nu_6$ resonance will have a final eccentricity in the range $e_{min}$ to $e_{max}$, where

$$e_{min,max} \simeq |e_i \pm \delta_e| \,, \tag{3}$$

and

$$\delta_e \equiv \left| \varepsilon \sqrt{\frac{\pi}{|\lambda|\sqrt{a}}} \right| . \tag{4}$$

## 3. Application to the main asteroid belt eccentricity distribution

By fitting the observed eccentricity distribution of large ($H \leq 10.8$) main belt asteroids to fictitious distributions obtained from the above analytical models, both the initial $e$ distribution of the main asteroid belt and the migration rate of Saturn (via $\lambda$) may be obtained. We find two solutions that are consistent with our observational set.

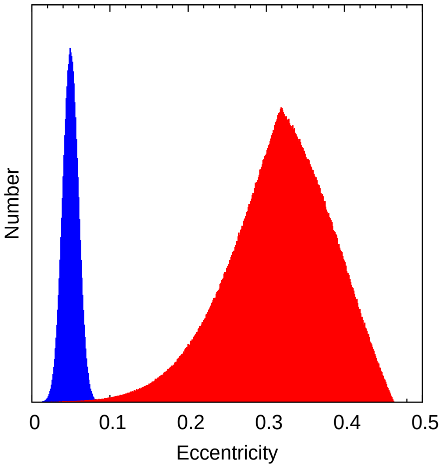

Figure 1: Initial conditions for the pre-$\nu_6$ resonance sweeping eccentricity distributions for main belt asteroids with semimajor axes $2.1$–$2.8$ AU. The blue histogram shows the“cold belt” solution, and the red histogram shows the “hot belt” solution (the y-axis is arbitrary).

Applying equation (3), we see that there are two possible solutions. In the first, $\langle e_i \rangle = 0.05$. This solution ($\delta_e = 0.14$) requires a migration rate for Saturn of $\dot{a}_6 = 4$ AU My$^{-1}$. We dub this solution the “cold belt” solution. The second solution exists if we consider that eccentricities in the main belt are restricted by the orbits of Mars and Jupiter on either side, such that stable asteroid orbits do not cross the planetary orbits. In this case, an initial single Gaussian eccentricity distribution with a mean greater than $\sim 0.3$ would be severely truncated, Applying equation (3), we find that $\delta_e = 0.21$ provides a good fit. The corresponding migration rate of Saturn is $\dot{a}_6 = 0.8$ AU My$^{-1}$. We dub this solution the “hot belt” solution (see Figure 1).

## 4. Summary and Conclusions

The two solutions obtained here for the pre-sweeping $e$ distribution of the main belt have two very different implications for models of the primordial excitation and depletion of the main asteroid belt. The “cold belt” solution implies that the asteroid belt became depleted without being very much excited. The “hot belt” solution requires a much greater level of eccentricity excitation than the observed, modern main belt suggests. Unfortunately, an analysis of eccentricity alone cannot distinguish between these two very different scenarios for the evolution of the solar system.

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