

Onset of Gravity-Capillary Waves on Titan's Lakes and Seas

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1. Introduction

The current lack of evidence for surface waves in Titan's polar regions is intriguing given the predominance of aeolian features in equatorial latitudes. This apparent discrepancy may be the result of observational bias in timing or location. If not, it may be caused by differences between the conditions required for wave generation on Titan and Earth. The process of initial wave generation on Titan was discussed by Lorenz et al. (2010). We extend the work of Lorenz et al. (2010) by investigating the effects surface tension (T), density (ρ), kinematic viscosity (ν), and gravity (g). In addition, we compare models of Bragg backscatter for water and liquid hydrocarbon and compare threshold friction velocities (u_*), which are related to wind stress ($\tau = \rho_{air} u_*^2$), to GCM predictions.

2. Wave Generation

Three common mechanisms for transferring energy from wind to short surface waves are the resonance mechanism (Phillips, 1957), the shear flow mechanism (Miles, 1957), and the Kelvin-Helmholtz (KH) mechanism (Kelvin, 1871).

The resonance mechanism excites waves through direct action of turbulent pressure fluctuations on the liquid surface (Phillips, 1957). It provides a broad-spectrum energy input and is associated with creating small-scale wavelets (sometimes referred to as "catspaw") (Schwartz and Marchello, 1968).

The shear flow mechanism neglects turbulent pressure fluctuations and provides focused wave generation through interaction with a parallel shear flow. Energy transfer is accomplished through inviscid Reynolds stress in a critical layer (Miles, 1957) and viscous Reynolds stress near the surface of a curved wind profile (Benjamin, 1959). The shear flow mechanism is attributed to the generation of small waves initially at the critical wavelength ($\lambda_{crit} = 2\pi\sqrt{T/(\rho g)}$) (Schwartz and Marchello, 1968). Models of the mechanism predict u_* of 4-5 cm/s (~ 1 m/s at 2 m height)

for air over water (Miles, 1957), which is in approximate agreement with ocean surface and long-fetch flume observations (Kahma and Donelan, 1988). Shtemler et al. (2008) argues that the threshold u_* is proportional to the minimum wave speed ($c_{min} = \sqrt[4]{4gT/\rho}$), suggesting values ~ 2 times smaller on Titan. Models of the shear flow mechanism disagree on the importance of kinematic viscosity (ν_{liq} & ν_{air}), ranging from independent to square root dependence, potentially modifying u_* by an additional factor of ~ 2 depending on liquid composition. More detailed analysis is required to understand the effects of viscosity and air density on the shear flow mechanism.

The classical KH theory considers a tangential discontinuity between uniform flows of two liquids (Kelvin, 1871). Miles (1959) updated the theory to account for logarithmic velocity profiles in the upper fluid (air). The KH mechanism, which is independent of ν_{liq} , predicts threshold u_* of ~ 50 cm/s for air over water, an order of magnitude larger than observed. However, the KH mechanism correctly predicts threshold u_* for both viscous and/or contaminated liquids, where small-scale ripples are damped by viscous dissipation (Miles, 1959). The KH mechanism is believed to be important for longer wavelength water waves at high wind speeds (Shtemler et al., 2008). The modified KH theory of Miles (1959) predicts threshold friction velocities of ~ 7 cm/s for liquid hydrocarbon on Titan.

Table 1: Model Parameters.

Parameter	Earth	Titan
ϵ_{liq}	53.98+34.38i	1.8+0.0007i
ρ_{air}	0.0012 g/cm ³	0.005 g/cm ³
ρ_{liq}	1 g/cm ³	0.6 g/cm ³
ν_{air}	0.154 cm ² /s	0.0126 cm ² /s
ν_{liq}	0.01 cm ² /s	0.005-0.03 cm ² /s
g	981 cm/s ²	135 cm/s ²
T	73 dynes/cm	18 dynes/cm
λ_{crit}	1.7 cm	3.0 cm
λ_{obs}	2.16 cm	2.16 cm

3. Predicted Friction Velocities

Figure 1 show the predicted u_* values from the GCM of Graves et al. (submitted). The results suggest that friction velocities do not exceed ~ 1 cm/s in the polar regions except during summer. Cassini has been observing Titan during northern winter and did not observe the southern lakes until Dec. 2007 (T39) at $L_s \sim 330^\circ$, when no waves are expected. However, u_* velocities during northern summer hover around the predicted threshold u_* values discussed above and suggest that surface waves may be generated in the northern seas during the Cassini Solstice mission.

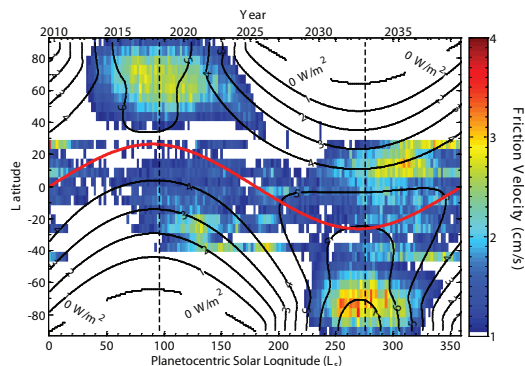


Figure 1: Daily averaged friction velocities (u_*) from Graves et al. (submitted) using a three dimensional idealized Titan GCM. Values represent the 95% quantile of the longitude distribution at a given latitude and time after averaging over 5 seasonal cycles.

4. Bragg Scattering Model

At moderate incidence angles ($10^\circ - 60^\circ$) the microwave backscatter of a wavy surface is dominated by Bragg scatter. Bragg scatter is modeled using a small perturbation model coupled to a wave power spectrum evaluated at the Bragg wavenumber ($k_b = 2k \sin(\theta)$) (Ulaby et al., 1982). Figure 2 shows a Bragg scattering model for the K_u -band normalized backscatter cross-section (σ_o) of Titan and Earth using the wave power spectrum derived by Fung and Lee (1982). Note that although Fung and Lee (1982) does allow for different gravity and fluid properties, the spectrum is semi-empirical and was derived at 1 bar. The decrease in dielectric constant (ϵ) between water and liquid hydrocarbon results in a 15-25 dB decrease in σ_o . This decrease is partially offset by the increase in spectral power at the Bragg wavelength, which is similar to the critical wavelength for liquid hydrocarbon on Titan. For u_* values near the shear flow mechanism

threshold, the model predicts σ_o in excess of -30 dB for $\theta < 27^\circ$. However, the power spectra of Fung and Lee (1982) has not been validated for $u_* < 12$ cm/s. Empirically on Earth, initial gravity-capillary waves are not observed in the K_u band until u_* is ~ 8 cm/s (Donelan and Pierson, 1987). On Titan, we may expect a similar relationship and not be able to observe the smallest range of gravity-capillary waves in off-axis backscatter. Nonetheless, waves generated from u_* of a few cm/s should be observable based on the increased spectral power at the Bragg wavelength.

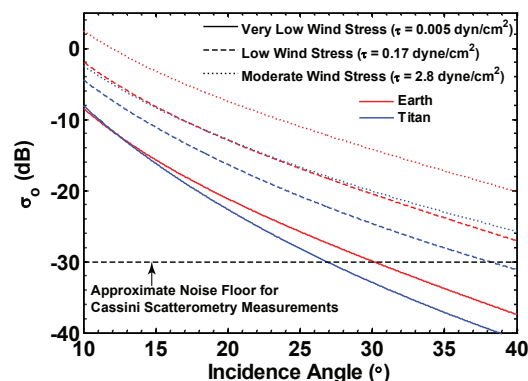


Figure 2: K_u -band Bragg backscatter model created using the Fung and Lee (1982) short-wave power spectrum. Very low wind stress corresponds to conditions around the onset of the shear mechanism ($u_* \sim 2$ cm/s), low wind stress corresponds to calm conditions ($u_* \sim 12$ cm/s), and moderate wind stress corresponding to a strong breeze ($u_* \sim 60$ cm/s).

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