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Determination of asteroids non gravitational perturbations

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Abstract

We discuss a method to determine non gravitational parameters of asteroids. Non gravitational perturbations can lead to secular effects, thus changing the dynamical evolution. Moreover, the probability of an impact between the Earth and an asteroid can significantly change. To succeed in our purpose, we add non gravitational parameters to the set of unknowns in the procedure of least squares orbital fit. This technique allow us to compute both nominal values and covariance matrices for these parameters, provided the observational arc is long enough. As a test, we apply this approach to the case of 1999RQ36.

1. Introduction

When dealing with asteroid dynamics, gravitational forces are the main terms driving their orbits. However there are cases in which also non gravitational components which could be relevant. Here we consider the following:

- The solar radiation pressure is given by the total transfer of linear momentum upon impact of the photons from the Sun on an orbiting object. For some simple shape objects the force due to the radiation pressure can be analytically computed, e.g., for a sphere we have

$$\mathbf{F} = -\frac{\phi_{\odot} A}{c}\hat{\mathbf{s}}\,, \qquad (1)$$

  where ϕ is the intensity of the energy flux, A is the effective cross section, c is the speed of the light and $\hat{\mathbf{s}}$ is the direction of the object-Sun vector. $\mathbf{F}$ is proportional to the A/M parameter, thus the radiation pressure mainly affect small objects. An asteroid with a rotation axis different from the orbital plane experiences a secular perturbation in the semimajor axis when the 2 hemispheres have different shape or different optical properties.

- The Yarkovsky effect is due to secular perturbations on the semimajor axis caused by the anisotropic emission of thermal photons. It is usually considered in relation to meteoroids or small asteroids (about 10 cm to 10 km in diameter), as its influence is most significant for these bodies. Over large time spans (millions of years), this force can move main-belt asteroids they reach a resonance, when gravitational perturbations can move them into the inner solar system. To give a quantitative estimate, a typical value for the drift on semimajor axis is 10^-4 AU/My, which leads to a large change over the age of asteroids.

2. Orbits and observations

The two essential elements of orbit determinations are orbits and observations. Orbits are solution of an equation of motion:

$$\mathbf{y} = \mathbf{f}(\mathbf{y}, t, \boldsymbol{\mu})\,, \;\; \mathbf{y}(t_0) = \mathbf{y}_0 \qquad (2)$$

where $\mathbf{y}$ is the state vector (e.g., position and velocity of an asteroid), $\boldsymbol{\mu}$ is a vector of dynamical parameters (e.g., geopotential coefficients, area to mass ratio) and f is the evolution law. If we knew the values of $\boldsymbol{\mu}$ and $\mathbf{y}_0$ we would be able to predict the value of $\mathbf{y}$ for each time t. In practice the situation is different, since we do not typically know these quantities, in particular the initial conditions. To solve the problem we need observations r_i, e.g. angular positions in the sky or radar measurements. Observations can be compared to the prediction function $R(\mathbf{y}, t)$ and since they are not generally coincident we have the residuals:

$$\xi_i = r_i - R(\mathbf{y}(t_i), t_i)\,. \qquad (3)$$

The residuals can be expressed as a function of the unknown parameter $\mathbf{x}$. As an example, take $\mathbf{x} = \mathbf{y}_0$, thus we assume a perfect knowledge of the dynamical law. For each value of the initial conditions we can compute the residuals by propagating the corresponding orbit.

3. Least squares fit

The minimum principle states that the "best value" for $\mathbf{x}$ is the one minimizing the target function

$$Q(\mathbf{x}) = \frac{1}{m}\boldsymbol{\xi}^T W \boldsymbol{\xi} \qquad (4)$$

where W is a definite positive weighting matrix. To find the minimum of Q we need to find $\mathbf{x}$ such that

$$\frac{\partial Q}{\partial \mathbf{x}} = \frac{2}{m}\boldsymbol{\xi}^T B = 0\ ,\ B = \frac{\partial \boldsymbol{\xi}}{\partial \mathbf{x}}\ . \qquad (5)$$

The solution can be find by using the differential corrections, an iterative procedure defined as follows:

$$\mathbf{x}_{k+1} = \mathbf{x}_k - (B^T B)^{-1} B^T \boldsymbol{\xi}(\mathbf{x})\ . \qquad (6)$$

An imperfect knowledge of the dynamic model due to non-gravitational perturbations, implies quadratic divergence of orbits computed with different models. If there are not physical measuring allowing us to directly compute non gravitational parameters, they can be determined by including them in the vector $\mathbf{x}$ of unknowns.

To succeed in our goal we need to take into account that there are constraints. First, the orbit of the object has to be very accurate since the effect we want to measure a comparatively small effect. Second, the problem has to be over determined enough, otherwise there could be a degeneracy when applying the differential correction procedure.

4. A test case: 1999RQ36

1999RQ36 is the most Potentially Hazardous Asteroids with very well known orbit. The probabilities accumulate to a total impact probability of approximately 10^{-3}, with a pair of closely related routes to impact in 2182 comprising more than half of the total. According to NeoDyS-2 the standard deviations of its Keplerian are the following:

$$\sigma_a = 1.191 \times 10^{-10}\ \text{AU}, \qquad \sigma_e = 2.889 \times 10^{-8},$$
$$\sigma_i = 3.721 \times 10^{-6}\ \text{deg}, \qquad \sigma_\Omega = 5.930 \times 10^{-6}\ \text{deg},$$
$$\sigma_\omega = 6.332 \times 10^{-6}\ \text{deg}, \qquad \sigma_M = 3.073 \times 10^{-6}\ \text{deg}.$$

1999RQ36 will be observable optically from August 2011 and may be a radar target in September. If radar observations are successful, the knowledge of the Yarkovsky non-gravitational effect can be improved by more than an order of magnitude. The method described above is used to achieve a better knowledge of the Yarkovsky effect and to update the impact probability.

5. Summary and Conclusions

We described a procedure to compute non gravitational parameters, such as radiation pressure and Yarkovsky effect. This was done by including non gravitational parameters to the set of unknowns during the orbital least squares fit. As a test, such procedure was successfully applied to the 1998RQ36 case.

References

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