

# Instability of acoustic-gravity waves and vortices in a nonadiabatic terrestrial atmosphere

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## Abstract

A continuously stratified model of nonadiabatic terrestrial atmosphere with taking into account the temperature profile is constructed in order to study a possibility of instability development of acoustic-gravity waves. It is shown that the existence of regions in the atmosphere where the instability conditions are satisfied is due to the cooperation of thermal flow of solar radiation, infrared emission of the atmosphere, water vapor condensation, as well as thermal conductivity. Dispersion surfaces for acoustic-gravity waves are constructed for the altitudes less than 130 km. We also discuss large-amplitude vortices observed in Earth's troposphere and their possible structure.

## 1. Instability of acoustic-gravity waves

We use a local frame of reference with axis  $x$  directed to the east, axis  $y$  to the north, and axis  $z$  pointed upward. Acoustic-gravity waves propagating in Earth's atmosphere are described by the following equations:

$$\frac{\partial \mathbf{V}}{\partial t} + (\mathbf{V}\nabla)\mathbf{V} = -\frac{\nabla P}{\rho} + \mathbf{g}, \quad (1)$$

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{V}) = 0, \quad (2)$$

$$\frac{\partial P}{\partial t} + (\mathbf{V}\nabla)P + \gamma P \operatorname{div} \mathbf{V} = \frac{P}{c_v \rho T} \nabla \mathbf{J}_c - \frac{P}{c_v \rho T} \nabla \mathbf{J}_a + \frac{P}{c_v \rho T} \nabla \mathbf{J}_k - \frac{P}{c_v \rho T} \nabla \mathbf{L}, \quad (3)$$

where,  $\mathbf{V}$  is the speed of air molecules,  $P$  is the pressure,  $\rho$  is the density of air,  $\mathbf{g}$  is the acceleration of gravity,  $\gamma$  is the ratio of specific heat,  $T$  is temperature, and  $c_v$  is specific heat capacity at constant volume. On the right-hand-side of the Eq. (3)  $\mathbf{J}_c$  describes the heat flux in the atmosphere due to the solar radiation,  $\mathbf{J}_a$  refers to the infrared emission,  $\mathbf{J}_k$  refers to the heat flux due to water vapor condensation, and  $\mathbf{L}$  characterizes heat flux due to the thermal conductivity. For transport and absorption of solar radiation we use the equation [2]:

$$\frac{\partial J_c}{\partial z} = \mu \rho J_c, \quad (4)$$

where absorption coefficient  $\mu$  is determined by  $J_{c3} = J_{c0} \exp(-\mu \bar{\rho} H_0)$ ,  $J_{c3}$  is the solar radiation flux near the planet's surface,  $J_{c0} = 1367 \text{ Wt/m}^2$  is the solar constant,  $\bar{\rho}$  is the averaged atmosphere's density, and  $H_0$  is the atmosphere's depth. Infrared heat deflux is calculated using the Stefan-Boltzmann law for the grey body. The heat flux due to the water vapor condensation is considered to decrease with the altitude and is given by  $\mathbf{J}_k = L_k \rho_{w0} \exp(-z/h_k) \mathbf{V}$ , where  $L_k = 2,5 \cdot 10^6 \text{ J/kg}$  is heat of evaporation,  $h_k = 0,67 \text{ km}$  in winter and  $h_k = 1,35 \text{ km}$  in summer,  $\rho_{w0} = 4,19 \cdot 10^{-3} \text{ kg/m}^3$ . Heat flux due to the thermal conductivity is calculated taking into account molecular and turbulent terms. The role of the molecular thermal conductivity is much weaker than that of the turbulent one up to the altitude of 100 km. We use experimental approximation of the turbulent thermal conductivity for different altitudes [1].

To obtain the dispersion equation we transform and linearize the set of Eqs. (1)-(4) complemented by the equation of state and search the solution in the form of  $\exp(z/2H_0)\exp(-i\omega t + ik_x x + ik_z z)$ , where  $H_0$  is the characteristic depth of the atmosphere which slightly varies for different altitudes. The instability conditions are satisfied if  $\text{Re}(\omega) > 0$  and  $\text{Im}(\omega) > \eta K^2$ , where  $K$  is a wave number. Then we study instability regions for different wave numbers. For  $k_x=0.003$  and  $k_z=0.006$  we obtain the altitude dependence shown in Fig. 1.

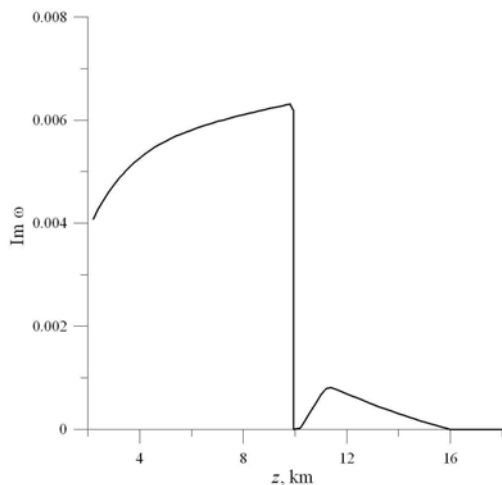


Figure 1: Growth rate vs. altitude.

We discuss also conditions in Titan's atmosphere to compare them with those on the Earth and to determine the instability regions. For the case of Titan's atmosphere one should take into account heat released in chemical reactions.

## 2. Vortices in the troposphere

In accordance with the results of the previous section (see Fig. 1) we conclude that acoustic-gravity waves can be unstable in the troposphere. This results in the formation of acoustic-gravity vortices in the troposphere. An example of such an acoustic-gravity vortex is given by Fig. 2. The speed and size of the vortex shown in Fig. 2 correspond to those observed in the troposphere [3].

## 3. Summary and Conclusions

We have studied the instability of acoustic-gravity waves in the atmosphere with taking into account

thermal flows of solar radiation, infrared emission of the atmosphere, water vapor condensation, as well as thermal conductivity. It has been shown that in Earth's atmosphere there are regions where the instability develops. The localized solutions in the form of vortices are obtained which correspond to those observed in Earth's troposphere. Analogies with Titan's atmosphere are discussed.

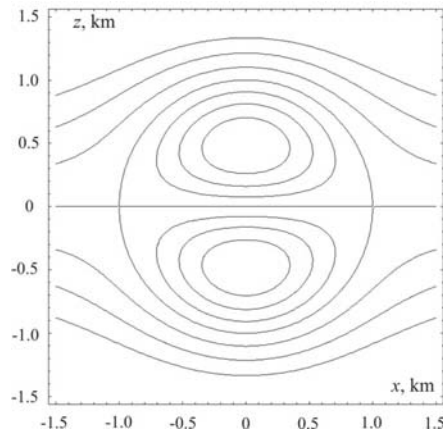


Figure 2: Streamlines of the vortex moving with the velocity of 30 km/h.

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