

Adhesion and collisional release of particles in dense planetary rings

A. Bodrova (1,2), J. Schmidt (2), F. Spahn (2) and Nikolay Brilliantov (3)

(1) Department of Physics, Moscow State University, Moscow, Russia (bodrova@polly.phys.msu.ru), (2) Institute of Physics and Astronomy, University of Potsdam, Potsdam, Germany, (3) Department of Mathematics, University of Leicester, Leicester, United Kingdom

We propose a simple theoretical model for aggregative and fragmentative collisions in Saturn's dense rings. In this model the ring matter consists of a bimodal size distribution: large (meter sized) boulders and a population of smaller particles (tens of centimetres down to dust). The small particles can adhesively stick to the boulders and can be released as debris in binary collisions of their carriers. To quantify the adhesion force we use the JKR theory [1]. The rates of release and adsorption of particles are calculated, depending on material parameters, sizes, and plausible velocity dispersions of carriers and debris particles. In steady state we obtain an expression for the amount of free debris relative to the fraction still attached to the carriers. In terms of this conceptually simple model a paucity of subcentimeter particles in Saturn's rings [2] can be understood as a consequence of the increasing strength of adhesion (relative to inertial forces) for decreasing particle size. In this case particles smaller than a certain critical radius r_{cr} remain tightly attached to the surfaces of larger boulders, even when the boulders collide at their typical speed. Furthermore, we find that already a mildly increased velocity dispersion of the carrier-particles may significantly enhance the fraction of free debris particles, in this way increasing the optical depth of the system.

The critical radius can be approximately expressed in the following form:

$$r_{cr} \sim \left(\frac{3}{5v_c^2} \right)^{3/10} \left(\frac{9\gamma R_{eff}}{4\rho_d} \right)^{1/2} \left(\frac{16\pi\rho_c D}{3} \right)^{1/5} \quad (1)$$

Here ρ_d and ρ_c are the material densities of debris and carriers, correspondently (we assume for ice $\rho_c = \rho_d = 900 \text{ kg/m}^3$), R_{eff} - the effective radius of carriers, v_c - velocity dispersion of carriers, $\gamma = 0.74 \text{ N/m}$ - the adhesion coefficient for smooth ice particles, which can be reduced in the case of rough particles. $D = \frac{3}{2} \frac{(1-\nu^2)}{Y}$, $Y = 7 \cdot 10^9 \text{ Pa}$ is the Young modulus and $\nu = 0.25$ - the Poisson ratio.

The dependence of the critical radius on velocity dispersion v_c and radii R of carriers is shown at fig. 1.

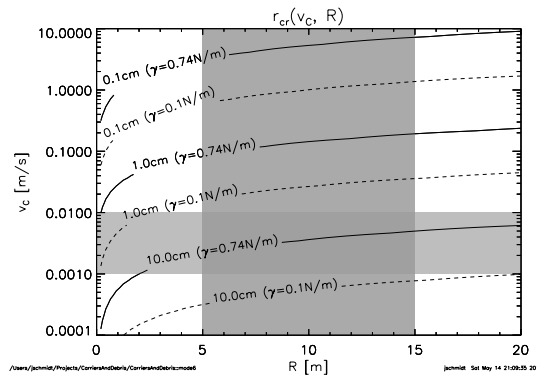


Figure 1: Contour graph, showing the dependence of the critical radius for debris release r_{cr} on velocity dispersion v_c and radii R of carriers. Solid contour lines are for the adhesion coefficient $\gamma = 0.74 \text{ N/m}$ and the dashed lines for $\gamma = 0.1 \text{ N/m}$. Parameter ranges plausible for Saturn's rings are highlighted in grey.

References

- [1] Johnson, K.L., Kendall, K., and Roberts, A.D.: Surface energy and the contact of elastic solids, Proc. Roy. Soc. London A, Vol. 324, pp. 301-313, 1971.
- [2] French, R. G. and Nicholson, P. D.: Particle Sizes Inferred from Stellar Occultation Data, Icarus, Vol. 145, pp. 502-523, 2000.