

Modeling the secular evolution of migrating planet pairs

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Abstract

The secular regime of motion of multi-planetary systems is universal; in contrast with the 'accidental' resonant motion, characteristic only for specific configurations of the planets, secular motion is present everywhere in phase space, even inside the resonant region. The secular behavior of a pair of planets evolving under dissipative forces is the principal subject of this study, particularly, the case when the dissipative forces affect the planetary semi-major axes and the planets move inward/outward the central star, the process known as planet migration. Based on the fundamental concepts of conservative and dissipative dynamics of the three-body problem, we develop a qualitative model of the secular evolution of the migrating planetary pair. Our approach is based on analysis of the energy and the orbital angular momentum exchange between the two-planet system and an external medium; thus no specific kind of dissipative forces is invoked. We show that, under assumption that dissipation is weak and slow, the evolutionary routes of the migrating planets are traced by the Mode I and Mode II stationary solutions of the conservative secular problem. The ultimate convergence and the evolution of the system along one of these secular modes of motion is determined uniquely by the condition that the dissipation rate is sufficiently smaller than the proper secular frequency of the system. We show that it is possible to reassemble the starting configurations and migration history of the systems on the basis of their final states and consequently to constrain the parameters of the physical processes involved.

1. Introduction

It is presently accepted that migration plays an important role in the dynamical history of planetary systems. Several mechanisms have been proposed to be responsible for planet migration; some of these are i) planet interactions with a protoplanetary disk of gas/dust, ii) gravitational scattering and clearing of remnant plan-

etesimal debris by the planets, iii) direct collisions between the planets, and iv) tidal interactions of the planets with a central star. There exists a vast literature on this issue; the reader is referred to Armitage (2010) for reviews of planet migration and references therein.

In this study, we propose a simple method for a qualitative study of the planet migration, which does not require a detailed knowledge of dissipative mechanism, its physical properties and starting configuration of the migrating system. This is because we do not consider any specific kind of dissipative forces (they can be originated by tidal torques in the disc, tidal interactions with the star, ejection of planetesimals by planets, direct collisions etc). Knowing that, under dissipation, the energy and the orbital angular momentum of the planet system are no longer conserved, we model the migration through the variation of the energy and parameters of the secular dynamics: the angular momentum and the semi-major axes ratio.

2. The dissipative secular evolution

We suppose that two planets, interacting secularly (the planets are far enough from any mean-motion resonance), undergo external perturbations which affect the energy and angular momentum of the system. One consequence of such perturbations is that the planet semi-major axes are altered and the planets move inward/outward the central star, the process known as planet migration. The variation of the energy over a small time increment is given by

$$\Delta \overline{\mathcal{H}} = \frac{\mu_1 m_1' \Delta a_1}{2a_1^2} + \frac{\mu_2 m_2' \Delta a_2}{2a_2^2} + \frac{k_2 m_1 m_2 \Delta D}{D^2}, \quad (1)$$

where Δa_1 , Δa_2 and ΔD are variations of the semi-major axes and the separation between the planetary orbits, respectively. The sum of the two first terms defines the change of the *orbital (Keplerian) energy* of the system, while the latter term defines the change of the *secular energy* of the system.

The secular energy is defined by mutual secular interactions of the planets and its dissipation affects

mainly the planet eccentricities provoking their damping oscillations. Even when only one planet is directly subjected to friction forces, the orbital angular momentum exchange between the planets affects the eccentricity of the other planet. We show that, if the dissipation is sufficiently weak and slow (adiabatic approach), the planets ultimately evolve into a nearly steady state defined by a stationary solution of the conservative problem, either Mode I or Mode II of motion (Michtchenko and Malhotra 2004), depending on the physical parameters of the system. In what follows, the already damped system continues to evolve under acting dissipative forces, tracing the family formed by the stationary solutions. We show that the final configuration of the system is defined by the balance between the orbital and secular energy variations during migration, together with the angular momentum exchange between the system and an external medium.

3. Figures

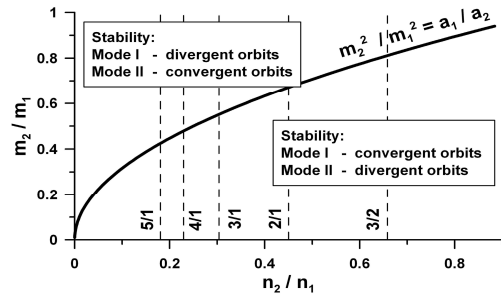


Figure 1: Domains of the stable centers defined by the condition $\sqrt{\frac{a_1}{a_2}} = \frac{m_2}{m_1}$ on the parametric $(n_2/n_1, m_2/m_1)$ -plane, where n is the mean motion of the planet. Locations of the main mean-motion resonances are indicated.

4. Summary and Conclusions

Analyzing the migration planetary tracks, we show that the evolution and the final state of the secular system, evolving under weak dissipative forces, depend on the balance between the orbital and secular energy variations, as follows:

- the loss of the orbital and secular energy defines the total circularization of two convergent orbits, allowing a capture in low-order mean-motion resonances;

- the loss of the orbital energy and the gain of the secular energy define the totally circularized orbits and the increasing mutual planetary distance;

- the gain of the orbital and secular energy defines the increasing mutual planetary distance, together with the increase of the eccentricities of the orbits;

- the gain of the orbital energy and the loss of the secular energy define the continuous increase of the eccentricities of two convergent orbits, when the secular system is broken due some external perturbations, such as mean-motion resonance interactions or close encounters between the planets.

Finally, we show several examples found among the planets of the Solar System and the known extra-solar systems, whose actual configurations could contain records of the dissipative evolution in the past, according to one of the above scenarios.

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