

Two New Codes to Simulate Narrow Eccentric Rings

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1. Introduction

In [1], we argued that sufficiently eccentric rings, such as those of Chariklo and Haumea, could self-confine for millions of years until their eccentricities damped below a critical value, using the results of [2] and [3]. To test this theory, we used the N-body integrator `epi_int` ([4]) to simulate Chariklo’s ring evolution. Unfortunately, `epi_int` suffered from unphysical secular eccentricity increases that made it unsuitable for tests of very long timescales, forcing us to compromise via simplified and contrived simulations. While our simulations agreed with predictions, their lack of rigor led us to seek alternative solutions.

Accordingly, we built a modified version of `epi_int`, dubbed `epi_int_lite`, and also upgraded our existing code, `HNBody` ([5]), to simulate narrow eccentric ring evolution. These new schemes do not suffer from `epi_int`’s unphysical eccentricity increases. We simulate Chariklo’s rings and will report on their stability using both of these new codes.

2. Numerical Integrators

2.1 `epi_int_lite`

`Epi_int_lite` is a new N-body code that evolves rings using a similar formalism to `epi_int`; rings are divided into a user-specified number of “streamlines,” upon which ring particles are initialized and then left free to wander. Like `epi_int`, `epi_int_lite` uses a drift-kick scheme. The main difference is that `epi_int_lite` only attempts accuracy to $O(e)$ for eccentricity e while `epi_int` attempts accuracy to $O(e^2)$.

Using this approximation, the drift step of the integration is straightforward; `epi_int_lite` converts the ring particles’ cylindrical coordinates to orbital elements, calculates the J_2 -corrected mean motion (see Section 3), updates each particle’s mean anomaly, and converts back to cylindrical coordinates. Since most rings have $e \ll 1$, the $O(e)$ approximation is usually sufficient. Furthermore, this modification to the `epi_int` algorithm eliminates the unphysical

eccentricity increases, allowing us to study the long-term stability of Chariklo’s rings, as desired.

2.2 Upgraded `HNBody`

Our first upgrades to `HNBody` were reported in [6], and our main result was that we could simulate viscous, self-gravitating rings using the same streamline formalism as `epi_int`. However, we used only one tracer particle per streamline to calculate its orbital elements, which we required in order to calculate gravity between streamlines. This (along with coding in C rather than IDL) yielded a speedup factor of ~ 130 , but only permitted us to simulate rings with mode $m = 1$, i.e., rings that were confocal ellipses. `HNBody` can also simulate fully 3D rings.

In reality, most rings are superpositions of multiple modes, which has limited the usefulness of our `HNBody` upgrade. Thus, we upgraded `HNBody` again to allow it to simulate a ring comprised of an arbitrary combination of any number of modes. With these modifications complete, we can now fully simulate Chariklo’s rings with `HNBody` as well as `epi_int_lite`, allowing us to test the two codes against each other.

3. J_2 -Corrected Orbital Elements

Both `epi_int_lite` and `HNBody` rely on extremely accurate conversions between coordinates and orbital elements, which they apply every step. The familiar standard osculating elements are formally only valid for unperturbed keplerian orbits. While such orbits only exist in theory, any perturbations are usually small enough to be safely ignored. Here, however, we require orbital elements that have been corrected to account for the perturbation due to the central body’s deviation from sphericity, typically parameterized via the familiar J_{2n} terms for $n = 1, 2, 3, \dots$

Currently, [7] hold the gold standard for such conversions, which produce so-called “epicyclic” elements, (claimed to be) accurate to $O(J_2 e^2, J_2 i^2)$ for eccentricity e and inclination i . However, in the limit that J_2 goes to 0, the epicyclic elements do not

correctly reduce to the osculating solution for a keplerian orbit, but rather to a truncated version accurate only to $O(e^2, i^2)$. This property is obviously undesirable, so we have derived our own J_2 -corrected elements that perform correctly for small J_2 . Using our algorithm ensures that we can safely simulate rings for hundreds of millions of years or longer without introducing artifacts due to conversion errors. We will compare our algorithm to that introduced in [7] (see also [8]), and note the advantages and disadvantages of each.

References

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