

Modelling the flow of thin and condensible atmospheres of icy planetary bodies

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1. Introduction

Fluid models of thin and condensible atmospheres are useful in understanding the dynamics of various planetary bodies and exoplanets. For tidally locked planets or planets with negligible spin, a 1D model allows for a relatively simple look at flow conditions within the atmosphere. Such a model was used to analyze the supersonic flow on the surface of Io [1] and was later used to study the atmospheres of tidally-locked lava-planets such as CoRot-7b and Kepler-10b [2]. In this study, we will apply the same technique to study the flow conditions of icy/snowball Earth-like exoplanets. We also expanded upon the model to include surface-atmosphere coupling of energy.

2. Methods

The 1D model used in this study stemmed from the shallow-wave equations considered in spherical coordinates [1]. The assumptions used here are negligible planet rotation and hydrostatically bound molecules. Another assumption is that since the atmosphere is turbulent and well-mixed, the pressure P , the temperature T , and the flow velocity V are treated as constant at within the atmosphere. As sublimation is the main forcing for the flow, the solutions are dependent on the surface temperature which is wholly a function of the angular distance (θ) from the subsolar point. The model involves 3 equations concerning the conservation of mass (Eq. 1), momentum (Eq. 2), and energy (Eq. 3) as follow:

$$\frac{1}{r \sin(\theta)} \frac{d}{d\theta} \left(\frac{V P \sin(\theta)}{g} \right) = m E \quad (1)$$

$$\frac{1}{r \sin(\theta)} \frac{d}{d\theta} \left(\frac{(V^2 + \beta C_p T) P \sin(\theta)}{g} \right) = \frac{\beta C_p T P}{g r \tan(\theta)} + \tau \quad (2)$$

$$\frac{1}{r \sin(\theta)} \frac{d}{d\theta} \left(\frac{(V^2/2 + \beta C_p T) V P \sin(\theta)}{g} \right) = Q \quad (3)$$

The boundary conditions are: flow velocity V is 0 at the subsolar point and the antisolar point, and $T(\theta=0)$ is the same as the surface frost temperature. The desired equilibrium-state solution is reached by finding an appropriately precise initial pressure at the subsolar point for the flow to reach supersonic speed at a critical angular distance θ_{crit} where a hydraulic jump occurs. A dynamically changing integrative step is taken into account for the instability of the system when approaching the critical point. Surface-atmosphere coupling is then introduced by allowing for energy exchanges between the two.

3. Results and Discussion

Looking at the basic Io case where there is no exchange between the surface and the atmosphere, the flow became supersonic at around $\theta_{crit} = 25^\circ$ where the wind speed reaches 124 m/s. The critical point occurs at a smaller θ than calculated by Ingersoll [1] where $\theta_{crit} = 34^\circ$; this is due to the change in how surface temperature is determined where geothermal heating (from under the surface) is accounted everywhere and not just on the night side in this study.

Coupling the surface and atmosphere where heat can be transferred between the two introduces more instability to an already sensitive system. When allowing only 60% of the energy for exchange, the critical angle also occurs at $\theta_{crit} = 25^\circ$ suggesting that the coupling process made a relatively small difference in the solution (Fig. 1).

When applying the model to an Earth-size planet with an ice-water covered surface where the temperature at the subsolar and antisolar point is 270 K and 100 K respectively, there is more instability compared

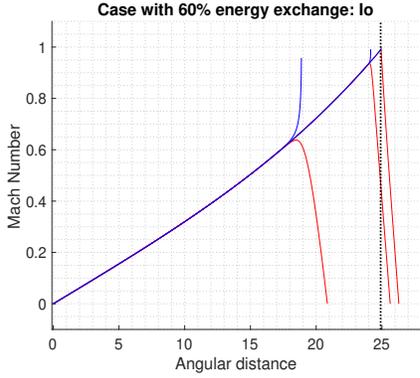


Figure 1: Figure shows the flow on Io with 60% exchange of energy. Instability in the model can cause the solution to exponentially expand (blue lines) or decay (red lines). Therefore, when both solutions start to diverge past a certain point, the integrative step will decrease allowing the solutions to approach θ_{crit} (black dashed line) in a more stable manner.

to the Io case. Without surface-atmosphere coupling, $P(\theta=0)$ is 450 Pa where θ_{crit} is expected to be around 32° - 35° (Fig. 2).

The sublimation rate E can be controlled by the molecular speed proportional to $\sqrt{k_B T/m}$. Setting this to a specified velocity v_c will allow us to see how the flow behaves under different sublimation rates. As expected, a smaller v_c will result in a slower flow where $v_c < 10^{-1} m/s$ will cause the flow to stay subsonic everywhere. This creates a thinner atmosphere with a maximum P of 29 Pa at the subsolar point and the maximum wind speed of 130 m/s is achieved at $\theta = 50^\circ$; the atmosphere will start to condense onto the surface at $\theta = 56^\circ$.

Acknowledgements

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References

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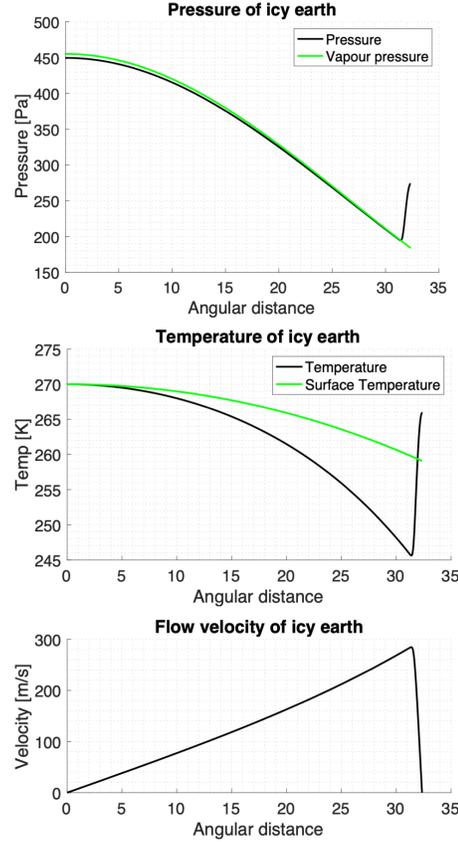


Figure 2: Figure shows the water vapour case without surface-atmosphere coupling. The solution here decays at around Mach number 0.9 at $\theta = 31.5^\circ$.

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