

# Stability Regions Associated to Equilibrium Points of Irregularly Shaped Bodies

**Othon Winter**, Tamires Moura, Gabriel Borders-Motta, Giulia Valvano, Rafael Sfair and André Amarante  
Grupo de Dinâmica Orbital & Planetologia, São Paulo State University - UNESP, Guaratinguetá, Brazil  
(othon.winter@unesp.br)

## Abstract

The shape of a small body (asteroid, comet, and satellite) and its rotational motion determine the existence of equilibrium points around it. The location and size of stability regions around small bodies provide information that can have different implications. For instance, it can indicate the locus of additional components (a moonlet or a cloud of natural debris), it can also be relevant for orbital design of a space mission (navigation and guidance). In the current work we compute the equilibrium points, the regions of stability associated to them and discuss their implications for a set of peculiar irregularly shaped small bodies.

## 1. Introduction

In general, asteroids, comets and small satellites present an irregular shape. Through radar observations and spacecraft imaging, the shape of small bodies has been mapped in terms of a polyhedron with several thousands of triangular faces. Assuming a constant density along the whole body, the gravitational potential can be computed through different approaches, as, for example, using the polyhedra model [1,2] or the mass concentration model - MASCONS [3,4]. Similarly to the restricted three-body problem, this system also has an integral of motion as the Jacobi constant, and from that can be obtained zero velocity curves/surfaces.

Many dynamical characteristics can be explored adopting such approaches. Taking into account the gravitational acceleration produced by the irregular shaped body and the centrifugal acceleration due to its rotation, one can compute the equilibrium points of this system. The number of points and their location varies enormously depending on its shape and rotation period. The linear stability of these points can be inferred from its eigenvalues. However, the nonlinearity of the problem might significantly affect the actual stability around those points that are linearly stable.

In the current work we are concerned with the actual stability in terms size and locations of stable region along a significant length of time.

## 2. Stable Regions

Considering a set of selected small bodies we explored their regions of stability associated to their equilibrium points. Some of the bodies studied are the main belt asteroid (216) Kleopatra, the comet 9P/Tempel and the larger body of the triple NEA 2001SN<sub>263</sub>.

Kleopatra is a relatively large asteroid (longest axis of the order of about 200 km) that has a peculiar "dog bone" shape, which was recently revised. The results adopting the "old" and the "new" shape do not change significantly. It presents a pair of almost symmetric points that are linearly stable, with their respective stable regions.

Comet Temple 1 main peculiarity is its very slow rotation (more than 40 hours), beyond a highly asymmetric shape. Slowly rotating bodies have equilibrium points significantly distant from their surfaces. In the case of Tempel is found a single stable equilibrium point far from its surface. The stability region around such point suggests the possibility that a cloud of natural debris might survive during the period of very low activity.

In the case of the Alpha body from the triple system 2001SN<sub>263</sub>, we have a small (less than 3 km diameter) fast rotator (about 3,4 hours) body. The computation indicated 13 equilibrium points, being one near its centre of mass and the remaining 12 close to its equatorial belt.

Figure 1 shows zero velocity curves at the equatorial plane of Alpha. The equilibrium points are indicated in red. As expected, the stable equilibrium points are intercalated between unstable ones. This structure of equilibrium points are certainly connected with the existence of a "belt" of material along the equator of this body

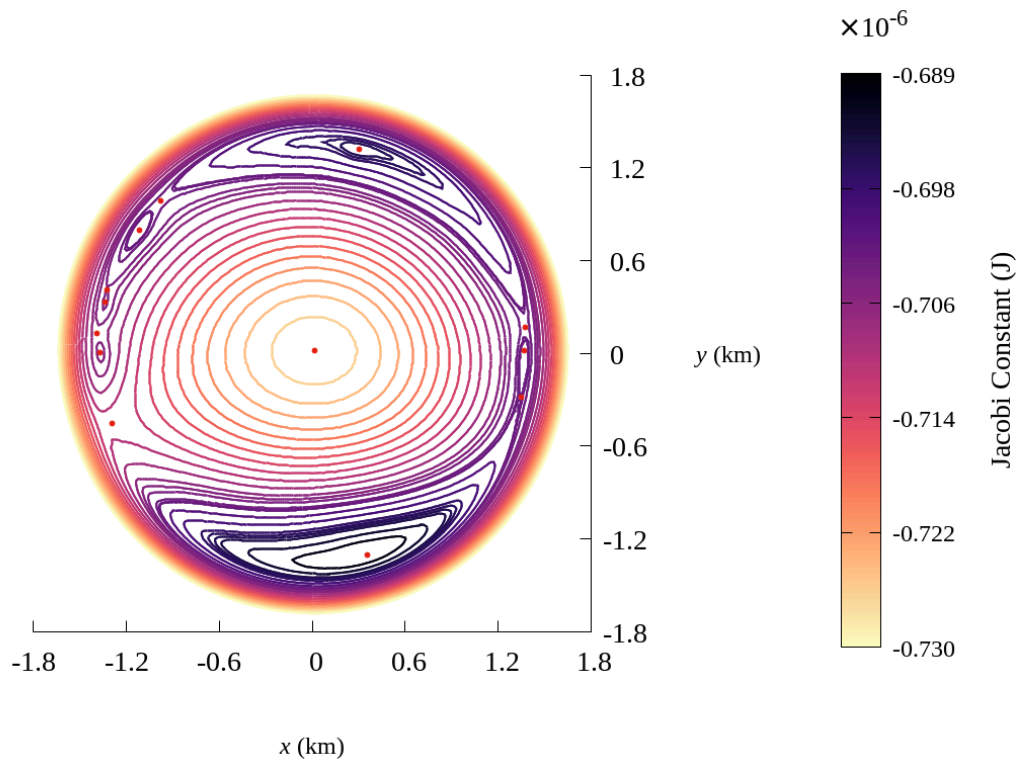


Figure 1: Zero velocity curves at the equatorial region of the main body (Alpha) of the triple system 2001 SN<sub>263</sub>. Note the equilibrium points indicated by red dots.

### 3. Summary

In this work are explored and analysed the equilibrium points and the associated stable regions of a set of peculiar irregular small bodies. More details will be given at the presentation.

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### References

[1] Werner, R. A.. The gravitational potential of a homogeneous polyhedron or don't cut corners. *Celestial Mechanics and Dynamical Astronomy*, 59, 253, 1994.

[2] Werner, R. A.; Scheeres, D. J.. Exterior gravitation of a polyhedron derived and compared with harmonic and mascon gravitation representations of asteroid 4769 Castalia. *Celestial Mechanics and Dynamical Astronomy*, 65, 313, 1996.

[3] Rossi, A.; Marzari, F.; Farinella, P. Orbital evolution around irregular bodies. *Earth, Planets, and Space*, 51, 1173, 1999.

[4] Borderes-Motta, G.; Winter, O. C.. Poincaré surfaces of section around a 3D irregular body: the case of asteroid 4179 Toutatis. *Monthly Notices of the Royal Astronomical Society*, v. 474, p.2452 - 2466, 2018.

[5] Chanut, T. G. G. ; Winter, O. C. ; Tsuchida, M.. 3D stability orbits close to 433 Eros using an effective polyhedral model method. *Monthly Notices of the Royal Astronomical Society*, v. 438, p. 2672-2682, 2014.

[6] Chanut, T. G. G.; Winter, O. C. ; Amarante, A. ; Araújo, N. C. S.. 3D plausible orbital stability close to asteroid (216) Kleopatra. *Monthly Notices of the Royal Astronomical Society*, v. 452, p. 1316-1327, 2015.