

# Retrieval of the fluid Love number $k_2$ in transit light curves: a feasibility study

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## Abstract

With about 4000 confirmed detected exoplanets, the characterization of their interior could potentially unveil information on their formation, migration, and habitability. The Love number  $k_2$ , when hydrostatic equilibrium of the interior is assumed, is an indication of mass concentration towards the body's center. Hence, it helps to further constrain the interior when combined with planetary mass and mean radius. We first summarize the planetary shape model which allows the retrieval of  $k_2$  from transit light curves. Second, we apply our model to synthetic data of WASP-121b and show that a precision  $< 90$  ppm/min is required to reliably retrieve  $k_2$  with present understanding of stellar limb darkening. Therefore we improve recent results based on ellipsoidal shape models.

## 1. Introduction

Knowledge of the planetary mass and mean radius is not sufficient to infer the interior structure, since the problem is degenerate with radial density profiles [1]. Hot Jupiters orbiting close to their Roche limit undergo strong tidal deformations. This modifies their shape from spherical to more complicated ones. Assuming hydrostatic equilibrium of the interior, the shape is a direct function of the fluid Love numbers  $k_j$ , of degree  $j$  [2]. In particular,  $k_2$  is an indication of mass concentration towards the body's center, providing additional information about the interior [3]. As a result of these deformations, the stellar eclipsed area during transit will differ from a transiting sphere, modifying the transit light curve. We briefly summarize the planetary shape model (Section 2), and apply it to synthetic data of WASP-121b (Section 3). This leads to constraints on the required noise level and limb darkening precision to reliably retrieve  $k_2$ .

## 2. Shape model

We assume a spherical star, a circular orbit, synchronous rotation, no interactions between rotation and tides, and absence of non-linear effects in the planetary response to perturbations. The radius at any surface point of colatitude ( $\theta$ ) and latitude ( $\phi$ ) is given by Equation (1) [2, 4].

$$r(\theta, \phi) = R_p \left( 1 + q \sum_{j=2}^4 h_j P_j(\lambda) \left( \frac{R_p}{d} \right)^{j+1} - \frac{1}{3} h_2 (1 + q) F_p^2 \left( \frac{R_p}{d} \right)^3 P_2(\cos \Theta) \right) \quad (1)$$

where  $R_p$  is the planetary mean radius,  $q$  is the mass ratio,  $h_j = 1 + k_j$  in the hydrostatic assumption,  $P_j$  are the Legendre polynomials of degree  $j$ ,  $d$  is the semi-major axis,  $F_p$  is the ratio between the orbital and rotational periods,  $\Theta$  is the obliquity, and  $\lambda$  is a geometrical factor.

## 3. Synthetic data

The parameters assumed are taken from [5,6] while an arbitrary value of  $k_2 = 0.5$  is chosen. We considered several white noise levels,  $\sigma$  (ppm/2min), reachable after 10 observed transits of WASP-121b with past, current and future observing facilities, summarized in Table 2.

Table 2: Considered white noise levels

Facility	$\sigma$ (ppm/2min)
JWST (NIRSpec)	23
Kepler	45
PLATO	63
CHEOPS	71
/	200
TESS	360

These white noise levels were randomly added to create synthetic light curves.

We retrieve the inclination, epoch, limb darkening, semi-major axis, planetary mean radius, and  $k_2$ .

Uniform priors were applied except for the stellar limb darkening coefficients, where two cases were considered: Gaussian priors with standard deviations ( $\sigma_{LDC}$ ) of 0.01 and 0.005. In doing so, we can compare our results to recently published performance with ellipsoidal shape models [7].

## 4. Results

We present in Figure 1 [8] the mean and standard deviation of the measured  $k_2$ , for both considered priors on the limb darkening coefficients. We also show the posterior distributions of  $k_2$  for all light curves realizations to assess the quality of the parameter estimation. The measured value must be precise and accurate to confidently say that the model can retrieve  $k_2$ . Thus, we require a precision of at least  $2\sigma$  and a relative error  $\leq 5\%$ .

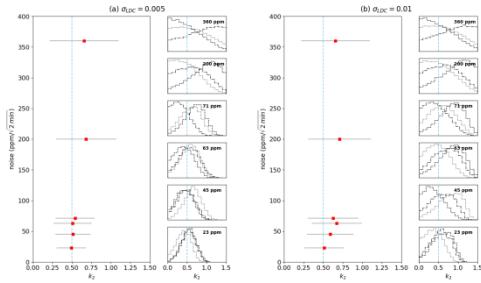


Fig 1. (a) Average values of the three realizations as a function of the noise level, and  $k_2$  posterior distributions, for  $\sigma_{LDC} = 0.005$ ; (b) Same as (a) but for  $\sigma_{LDC} = 0.01$ .

For a well constrained stellar limb darkening, we get a least a  $2\sigma$  detection with a relative error  $< 5\%$  for noise levels up to 63 ppm/2min. At 71 ppm/2min we also obtain a  $2\sigma$  detection of  $k_2$ , but with a relative error of about 9%. For higher noise levels, the relative error drastically drops and the posterior distributions of  $k_2$  widen and flatten, covering the whole physical range [0; 1.5] (see Figure 1(a)). When the accuracy on the limb darkening coefficients decreases (Figure 1(b)), we are able to reliably recover  $k_2$  with a noise level of 23 ppm/2min only. For higher noise values, the precision and relative error decidedly decrease.

## 5. Summary and Conclusions

The proposed three-dimensional shape model allows direct fitting of the true planetary mean radius, and fluid Love number  $k_2$ . Considering the close-in hot Jupiter WASP-121b as a test case, we showed that a noise level  $\leq 65$  ppm/2min (equivalently 90 ppm/min) and a standard deviation  $< 0.01$  on the limb darkening coefficients are required to reliably retrieve  $k_2$ . We thereby improve the performance of the three-axis ellipsoidal shape models by almost a factor 2. A careful treatment of noise sources is critical to achieve reliable measurements of  $k_2$ , and any improvement on stellar limb darkening would increase the performances summarized above. Such measurements would allow to further constrain exoplanetary internal structures by comparing the measured  $k_2$  to theoretical interior model expectations.

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