

Dust-ice two layer model for volatile Sublimation and Condensation on Icy Small Bodies

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Abstract

Icy small bodies can be generally described as a "dust-ice" t-wo layer system. The existence of the on-top dust mantle can significantly influence the sublimation and condensation of ice buried below. Here we discuss which kind of sublimation model could be used when the dust mantle has different thickness on different type of icy small bodies, like JFCs, MBCs and Extinct Comets.

1. Introduction

We have known that there exist lots of icy bodies beyond the "snow line" of the solar system, like KBOs that supplies so-called Jupiter Family Comets (JFCs), and the much more distant Oort Clound that supplies the Halley-family and long-period comets. Besides, [2] discovered a new population of comets in the main belt, named as main-belt comets (MBC-s), indicating that there still exist some icy bodies in the main belt.

When the stable orbit of a icy body is changed, making it moves to a certain distance to the Sun, where the surface temperature gets high enough for the sublimation of ice, near-surface ice on the icy body will sublimate, generating gas and escaping from the surface. The generated gas would probably blow away surrounding small dust particles, thus forming shining coma and cometary tail, then the icy body becomes a comet. However, some larger dust particles would be left behind and accumulate on the surface, thus gradually forming a dust mantle. As time goes on, the dust mantle would grow more and more thicker, making the outflow gas more and more weaker, until the time that no sufficient dust particles can be blown away to form observable dust coma, then we may define the comet to be an "Extinct Comet".

2. Model Description

2.1. Dust-ice two layer system

As a first approximation, let's describe an icy body to be dustice two layer system showed in Figure 1, with a dust mantle covered on the homogeneous interior of dust-ice mixture grains. Each dust-ice grain has a dust grain with radius b, density $\rho_{\rm d}$ and ice/dust mass ratio χ_0 (pure ice density $\rho_{\rm i}$). The dust mantle has porosity ϕ , tortuosity ς , and thickness $h_{\rm i}$ (ice front depth).

2.2. Sublimation of surface ice

In the case, ice is exposed to the surface, meaning that the dust mantle thickness

$$h_i \to 0$$
,

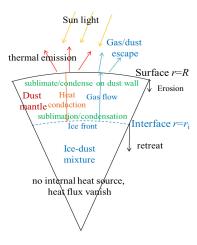


Figure 1: Dust layer on top of the ice-dust interior of an icy small body.

like typical fresh JFCs, the sublimation model can be significantly simplified by taking the assumption:

Once the solar insolation is strong enough to heat the surface ice to get a high enough temperature for sublimation, heat conduction between the surface ice and nearby materials can be ignored.

Then the sublimation temperature of the surface ice can be easily estimated from the surface energy conservation condition.

$$(1 - A_{\text{eff,B}})L_{\text{s}}\psi = \varepsilon_{\text{eff}}\sigma T_i^4 + \Delta H(T_i)\hat{m} \cdot \frac{1}{4}\tilde{v}_{\text{th}}n_{\text{E}}$$
, (1)

where $A_{\rm B}$ is the Bond albedo, ε is the average thermal emissivity, $\tilde{v}_{\rm th}$ is the average thermal velocity, $\Delta H(T_i)$ is the enthalpy of sublimation, $n_{\rm E}=P_{\rm E}(T_i)/k_{\rm B}T_i$ is the saturation number density and $P_{\rm E}$ is the saturation pressure that can be be calculated from the temperature via the integral form of Clausius-Clapeyron equation.

2.3. Sublimation of buried ice

In this case, the thickness of the dust mantle thickness h_i by far exceeds the seasonal thermal skin depth $l_{\rm st}$

$$h_i \gg l_{\rm st} \sim 1 \text{ m},$$

like "Extinct Comets", for exmaple (3200) Phaethon [3], the model can be well simplified by taking the following assumptions:

- (1) Although the surface temperature varies periodically due to rotation and orbital movement, the subsurface temperature below several seasonal thermal skin depths will be in a quasi-static equilibrium with the average solar insolation and thermal emission from the surface.
- (2) The heat flow in the dust mantle from the surface to the ice sublimation front is mainly consumed by the latent heat of sublimation there.
- (3) The difference between the subsurface dust mantle temperature $\tilde{T_0}$ and the temperature T_i of the ice front can be considered constant. This temperature difference continuously drives sublimation at the ice front.

The equilibrium subsurface dust mantle temperature \tilde{T}_0 can be estimated by assuming radiation equilibrium at the surface

$$\tilde{T}_0 = \left\lceil \frac{(1 - A_{\rm B})\tilde{F}}{\varepsilon \sigma} \right\rceil^{1/4},\tag{2}$$

where \tilde{F} is the annual average solar insolation.

To calculate the temperature T_i at the ice front $(r = r_i)$, we start with the energy balance equation at the ice front (in thermal equilibrium)

$$\nabla \cdot \vec{q} = Q = -\Delta H \hat{m} J \xi, \tag{3}$$

from which we obtain

$$q_{\rm i}(r_{\rm i}) = -\Delta H \hat{m} J,\tag{4}$$

which implies that the heat flow at the ice front is consumed by ice sublimation. For the heat flow at radius r in the dust mantle, energy conservation further gives

$$q = -\kappa \frac{\mathrm{d}T}{\mathrm{d}r} \approx \frac{q_{\mathrm{i}}r_{\mathrm{i}}^{2}}{r^{2}}.$$
 (5)

We assume that the sublimation flux J diffuses from the ice front to the surface. From mass conservation, we find the gas flow flux at a radial distance r to be

$$j = -\beta \frac{\mathrm{d}n}{\mathrm{d}r} \approx \frac{Jr_{\mathrm{i}}^2}{r^2},\tag{6}$$

where β is the Knudsen diffusion coefficient.

Integrating both sides of Equation (5) and Equation (6), we obtain

$$\begin{split} T \Big|_{r=R}^{r=r_{\rm i}} &\approx \int_{R}^{r_{\rm i}} \frac{q_{\rm i}}{\kappa} \mathrm{d} \left(\frac{1}{r} \right), \\ n \Big|_{r=R}^{r=r_{\rm i}} &\approx \int_{R}^{r_{\rm i}} \frac{J}{\beta} \mathrm{d} \left(\frac{1}{r} \right), \end{split}$$

which further gives.

$$T_{\rm i} - \tilde{T}_0 \approx r_{\rm i} \left(1 - \frac{r_{\rm i}}{R} \right) \frac{q_{\rm i}}{\tilde{\kappa}},$$
 (7)

$$n_{\rm i} - n_0 \approx r_{\rm i} \left(1 - \frac{r_{\rm i}}{R} \right) \frac{J}{\tilde{\beta}},$$
 (8)

assuming $\tilde{\kappa}$ and $\tilde{\beta}$ to be averaged constant in the dust mantle. For small bodies, the gravity is too low to prevent thermal escape of the water vapor molecules at the surface, thus we

$$n_0 \approx 0$$
.

And at the ice front, because of sublimation,

$$J = \frac{1}{4} \tilde{v}_{\rm th}(T_{\rm i})(n_{\rm E}(T_{\rm i}) - n_{\rm i}),$$

from which we obtain using Equation (7) and (8)

$$n_{\rm i} = \frac{r_{\rm i} \left(1 - \frac{r_{\rm i}}{R}\right)}{r_{\rm i} \left(1 - \frac{r_{\rm i}}{R}\right) + 4\beta/\tilde{v}_{\rm th}(T_{\rm i})} n_{\rm E}(T_{\rm i}), \tag{9}$$

$$J = \frac{\beta n_{\rm E}(T_{\rm i})}{r_{\rm i} \left(1 - \frac{r_{\rm i}}{R}\right) + 4\beta/\tilde{v}_{\rm th}(T_{\rm i})}.$$
 (10)

Moreover, combining Equation (4), Equation (7) and Equation (8), we find

$$\tilde{T}_0 = T_{\rm i} + \frac{\Delta H \hat{m} \tilde{\beta} n_{\rm i}(T_{\rm i})}{\tilde{\kappa}}, \tag{11}$$

which enables us to calculate the temperature T_i of the ice front from the equilibrium subsurface dust mantle temperature \tilde{T}_0 , as shown in Figure 2 for water ice, which can serve as the two limits of sublimation temperature of water ice on different icy small bodies.

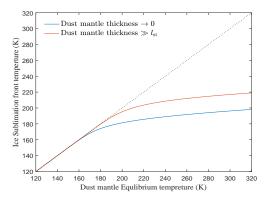


Figure 2: The relationship between the temperature T_i of the ice sublimation front and the dust mantle equilibrium temperature \tilde{T}_0 .

2.4. Sublimation of near-surface ice

In the case that there exist a dust mantle, but the dust mantle is so thin that the thickness is even smaller than the diurnal or seasonal thermal skin depth:

$$0 < h_i < \sim l_{\rm st},$$

the sublimation and condensation process would become much more complicated, probably including recondensation in the near-surface dust mantle, gas outburst at diurnal or seasonal terminator, and even moving surface boundary at particular time or location. The newly discovered MBCs may be like such case. And we are still working on developing both analytical and numerical methods to solve this problem.

References

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