

Binary Asteroid Formation: Applications of the Restricted Full Three-Body Problem

Alex B. Davis, Daniel J. Scheeres
 University of Colorado Boulder, Colorado, USA, (alex.b.davis@colorado.edu)

Abstract

We analyze the fate of ejected particles during a simulated binary asteroid formation event using the Restricted Full Three-Body Problem (RF3BP). The RF3BP consists of two rigid mass distributions behaving under the influence of their mutual gravity force and torques and a third massless particle under the gravitational influence of both bodies. We initialize the asteroids at the point of fission with massless particles ejected around the point of contact between the two bodies. Monte Carlo simulations of the dynamics provide insight into the effect of particle ejection state on their fate after the asteroids settle. The analysis is applied to binary asteroids 1999 KW4 and 65803 Didymos.

1. Introduction

Asymmetric binary asteroid formation occurs when an asteroid is spun up by YORP and other processes until reaching a critical spin rate and experiencing a mass shedding event; the ejected mass then forms the secondary asteroid. This process has been explored in work by Walsh, Richardson and Michel as well as Scheeres and Jacobson[1][2][3]. While these studies analyze the behavior of the secondary asteroid as it forms and settles into orbit, they ignore the dynamics of additional ejecta detached during the fission event. Using the inertia integral implementation of the Full Two-Body Problem (F2BP), described in Hou 2016, we have developed a computationally efficient implementation of the RF3BP [4]. The RF3BP models the primary and secondary asteroids as two gravitating rigid mass distributions and a third massless particle moving under their influence. Through this analysis we provide insight into the behavior and fate of ejecta during asymmetric binary formation.

2. Dynamics Model

The RF3BP is modelled by treating the third body as a massless particle behaving under the influence of two rigid mass distributions, Fig.1.

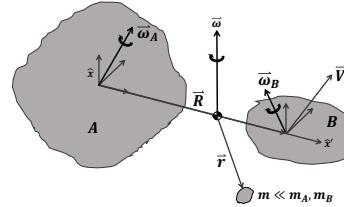


Figure 1: Full system diagram of the RF3BP.

Because the third body is massless, the dynamics are modelled as two processes. The motion of the mass distributions is modelled with the F2BP, which must account for the gravity interaction between each mass element making up the primary and secondary; resulting in a coupled gravity force and torque between the two bodies. Our implementation models the relative dynamics of the two bodies, approximating their mass distributions with inertia integrals[4].

$$T^{l,m,n} = \frac{1}{MR^{l+m+n}} \int_B x^l y^m z^n dm \quad (1)$$

Where l , m , and n sum to the truncation order N . The equations of motion and mutual gravity potential can be found in Hou 2016. Given the computed motion of the primary and secondary, the motion of a third particle is modeled in the rotating frame of the two asteroids; mapped from the inertial frame with the rotation matrix

$$\mathbf{C} = [\hat{R}, \hat{\omega} \times \hat{R}, \hat{\omega}] \quad (2)$$

Where \hat{R} is the relative position of the primary and secondary's center of mass and $\hat{\omega}$ is the orbital angular

velocity of the two bodies, computed as

$$\vec{\omega} = \frac{\vec{R} \times \vec{V}}{|\vec{R}|^2} \quad (3)$$

Because $\vec{\omega}$ is not constant in direction or magnitude, the particle's equations of motion are modified from the classical Circular Restricted Three-Body Problem (CR3BP) as[5]

$$\ddot{\vec{r}} = -\dot{\vec{\omega}} \times \vec{r} - 2\vec{\omega} \times \dot{\vec{r}} - \vec{\omega} \times (\vec{\omega} \times \vec{r}) - \vec{A} \quad (4)$$

Where \vec{r} is the position of the particle relative to the system center of mass and \vec{A} is the gravity acceleration experienced by the particle from the asteroid

$$\vec{A} = -\frac{\partial U_{13}}{\partial \vec{r}_{13}} - \frac{\partial U_{23}}{\partial \vec{r}_{23}} \quad (5)$$

The asteroid spin states are tracked in the potential, U_{i3} , by rotating the inertia integrals from their body fixed frames into the rotating frame.

3. Methodology

For our analysis we select two well-characterized asymmetric binary asteroids, 1999 KW4 and 65803 Didymos, and simulate their fission. We assume the bodies are rigid, constant density, and match current radar shape models. The simulation is initialized with the primary and secondary asteroids principal axes aligned and nearly in contact, described as the inner unstable relative equilibrium in Scheeres 2006[6]. A set of massless particles are initialized with small perturbations from the primary and secondary's closest point and simulated in parallel with the asteroid motion, as illustrated in Fig.2.

Monte Carlo analyses of the particle initial states are run over year long periods for both KW4 and Didymos. The results are statistically analyzed to help understand the fate of particles during an asteroid fission event. Of interest here are the fraction of ejected mass which leaves and remains in the system as well as the final resting place of particles returning to the primary or secondary. Such information would provide insight into binary evolution and potential areas of mass concentration which may experience land slides, or other redistribution events over the binary's lifetime

Acknowledgements

This material is based upon work supported by the National Science Foundation Graduate Research Fellow-

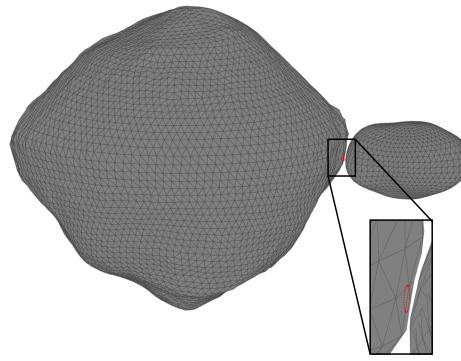


Figure 2: Example of initial geometry of particles, red, in the KW4 system, grey.

ship Program under Grant No. DGE 1650115. Any opinions, findings, and conclusions or recommendations expressed in this material are those of the author(s) and do not necessarily reflect the views of the National Science Foundation. Support from the DART project office is acknowledged via a JHU-APL grant.

References

- [1] Walsh, K.J., Richardson, D.C. and Michel, P.: Rotational breakup as the origin of small binary asteroids, *Nature*, Vol. 454, pp. 188-191, 2008
- [2] Scheeres, D.J.: Rotational fission of contact binary asteroids, *Icarus*, Vol. 189, pp. 370-385, 2007
- [3] Jacobson, S.A. and Scheeres, D.J.: Dynamics of rotationally fissioned asteroids, *Icarus*, Vol. 214, pp. 161-178, 2011
- [4] Hou, X., Scheeres, D.J. and Xin, X.: Mutual potential between two rigid bodies with arbitrary shapes and mass distributions, *Celestial Mechanics and Dynamical Astronomy*, pp. 1-27, 2016
- [5] Bellerose, J.A. and Scheeres, D.J.: Restricted Full Three-Body Problem: Application to Binary System 1999 KW4, *Journal of Guidance, Navigation, and Control*, Vol. 31, pp. 162-171, 2008
- [6] Scheeres, D.J.: Relative equilibria for general gravity fields in the sphere-restricted full 2-body problem, *Celestial Mechanics and Dynamical Astronomy*, Vol. 94, pp. 317-349, 2006