

Secular evolution of resonant small bodies' orbits: Kozai-Lidov cycles and adiabatic chaos

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Abstract

We study secular effects in orbital dynamics of minor bodies in mean-motion resonance within restricted three-body problem. Via double numerical averaging we construct evolutionary equations which allow to study the dynamics in detail without imposing any restrictions on the values of orbital parameters. This method also allows to describe the stochastisation mechanisms, which in particular drive the qualitative changes in secular dynamical regimes on a very large timescales.

Mean-motion resonances

Using a system “star-planet-asteroid” as an example mean-motion resonance (MMR) $p : (p + q)$ can be defined as orbital configuration in which asteroid completes p rotations around the star in the same amount of time the planet completes $p + q$ rotations (p and q are integer numbers). We focus our attention on exterior first-order MMRs ($q = 1$). In Solar system these resonances are represented in particular by plutino and twotino – large populations of Kuiper belt objects, which are in $2 : 3$ ($p = 2$) and $1 : 2$ ($p = 1$) MMRs with Neptune respectively.

Semi-analytical model

One of the key features of our model is the applicability to orbits with arbitrary values of eccentricity and inclination. It was made possible by a combination of the approach developed by Wisdom [1] with semi-analytical methods based on numerical averaging of disturbing function. The results reveal the possible scenarios of secular evolution, including a variety of Kozai-Lidov cycles, some of which appears to be new and unique.

Chaotic dynamics

Previous studies of secular dynamics in MMR relied on integrable models [2, 3], i.e. they did not include any chaotic dynamics. Our model allows studying the chaos in the system, with a focus on stochastisation mechanism, which is known as adiabatic chaos [4]. While the existence of chaos in first-order MMRs was indicated by numerical simulations [5], it has not received a proper attention in the past from theoretical point of view.

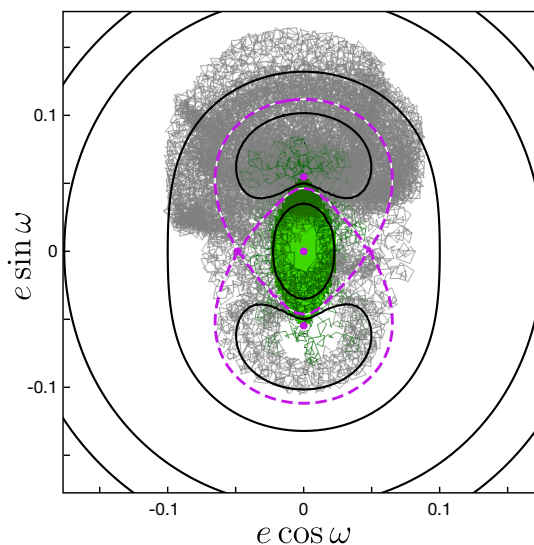


Figure 1: Numerical simulation vs analytical model in secular evolution of Kuiper belt object 2007JH₄₃. e and ω are the eccentricity and argument of pericenter respectively.

Comparison with numeric simulations of real objects' evolution

Many specific cases of Kozai-Lidov cycles, as well as mixing in stochastic layer and some other dynamical effects predicted by our model, are illustrated by examples from dynamics of known Kuiper belt objects.

Figure 1 gives an example of secular evolution in 2 : 3 MMR. The results of numerical simulation roughly follow the lines of analytical model, while also being under the effect of chaotic diffusion.

References

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