EPSC Abstracts
Vol. 13, EPSC-DPS2019-1634, 2019
EPSC-DPS Joint Meeting 2019

EPSC-DPS Joint Meeting 2019
15–20 September 2019 | Geneva, Switzerland

Atmospheric radio occultations using ray-tracing techniques

A. Bourgoin, M. Zannoni, L. Gomez Casajus, and P. Tortora
(1) Dipartimento di Ingegneria Industriale, University of Bologna, via fontanelle 40, Forlì, Italy
(adrien.bourgoin@unibo.it)

Abstract

In the context of radio occultation experiments, we discuss the benefits of a new formalism based on a full reformulation of the fundamental equations of geometrical optics. We use this formalism in order to analytically derive observable effects induced by the presence of winds in an oblate axisymmetric atmosphere. We assess the accuracy of the so-obtained solution by comparing it with outputs of a numerical integration of the fundamental equations of geometrical optics. Finally, we discuss applications of the formalism to real atmospheric radio occultations data processing, in the context of the Cassini and the JUICE space missions.

1. Introduction

Numerical ray-tracing (numerical integration of the fundamental equations of geometrical optics) has proven to be a powerful tool for processing data of radio occultations by oblate axisymmetric atmospheres with zonal winds [1]. If this technique is nowadays the most generic and the most accurate, it presents two disadvantages. Firstly, it does not provide a comprehensive description of how the atmosphere oblateness nor the zonal winds influence the light path and the light time. Secondly, because the initial pointing of the S/C's antenna has to be determined iteratively, the computation time is usually important.

To overpass these two difficulties, a purely analytical approach (analytical ray-tracing) has been recently proposed [2]. This method is based on a full reformulation of the fundamental equations of geometrical optics into a set of osculating equations which are similar to perturbation equations of celestial mechanics. This set of equations describes how the constants of integration of a reference solution, called the "hyperbolic solution", change according to variations in the refractivity profile of the atmosphere.

2. General assumptions

Assuming that the atmosphere is in hydrostatic equilibrium, it can be shown [3] that the density of the refractive medium is function of a generalized potential, Φ. In addition, in the context of elementary theory of dispersion [4], it is shown that surfaces constant value of the index of refraction fits surfaces of constant density value. Therefore, as it is done in [1, 2], we can assume that the expression of the index of refraction satisfies $n = n(\Phi)$. Finally, in order to not restrict ourselves to any particular function, an infinite series is considered, such that

$$n(\Phi) = \eta + \sum_{k=1}^{+\infty} \frac{(-1)^k}{k!} \alpha_{(k)} \Phi^k, \qquad (1)$$

where the generalized potential is given by

$$\Phi = U_0 + \sum_{l\geq 2} U_l - \Phi_C + U_{\text{tide}}(t). \qquad (2)$$

η is the value of the index of refraction at infinity, $\alpha_{(k)} \equiv -\mathrm{d}^k n/\mathrm{d}\Phi^k$, U_0 is the Newtonian gravitational potential, U_l is the non-spheric contribution to the gravitational potential, Φ_C is the centrifugal contribution, and $U_{\text{tide}}(t)$ is the tidal contribution.

3. Analytical ray-tracing

The reference solution is found for a very peculiar refractivity profile. Assuming first, that $\Phi = U_0$ (the index of refraction becomes spherically symmetric), then, that $\mathrm{d}n/\mathrm{d}\Phi = \text{const}$, we demonstrated [2] that the fundamental equation of geometrical optics possesses exact solutions for the light path (the shape of the trajectory is an hyperbola), the light time or the refractive bending. For instance, the light path describes an hyperbola and the expression of the hyperbolic index of refraction is given by

$$n_0 = \eta - \alpha_{(1)} U_0, \qquad (3)$$

with the convention $U_0 < 0$. The solutions are expressed in term of several arbitrary constants of integration characterizing the shape of the hyperbola, its spatial orientation, and a traveled length or a light time along the trajectory. By varying the constant of integration, we have been able to reformulate the fundamental equation of optics into a set of osculating equations describing the constants changes following any variation in the dependency of the refractive profile [2].

Using mathematical method of perturbation theory, we found approximate solutions for each contributions in Eqs. (1) and (2). We assessed the accuracy of the solutions by comparing the results with outputs of a numerical ray-tracing. For strong refractivity ($n - \eta = 10^{-4}$) and for typical values of Jupiter or Saturn's oblateness ($J_2 = 10^{-2}$), the analytical expression for the total light time (hyperbolic plus non-hyperbolic contributions) is accurate at the level of one part in 10^8.

4. Summary and Conclusions

The establishment of the osculating equations for the ray propagation has two main advantages. Firstly, it provides a simple geometrical picture for interpreting the path of the light ray. Secondly, it allows to solve analytically and systematically the equation of geometrical optics for non-radial dependencies.

In the context of real data processing, such an analytic solution could be either an ally or even a substitute to the purely numerical ray-tracing. Indeed, numerical ray-tracing demands an iterative solving of the pointing problem, which requires a large amount of computation time. The analytical ray-tracing could provide directly an accurate pointing direction, such that one or two iterations only would be sufficient.

At the same time, considering that the analytical ray-tracing is able to solve the path of the light ray or the light time, for any kind of refractivity dependencies and with high accuracy, we can think of a purely analytical procedure to process radio atmospheric occultations data. Such procedure will be compared with outputs of a purely numerical method in the context of Jupiter or Saturn's oblate axisymmetric atmosphere considering zonal winds.

Acknowledgements

We are grateful to the University of Bologna and to the Italian Space Agency (ASI) for financial support through the agreement 2018-25-HH.O in the context of ESA's JUICE mission.

References

[1] P. J. Schinder, F. M. Flasar, E. A. Marouf, R. G. French, A. Anabtawi, E. Barbinis, and A. J. Kliore: A numerical technique for two-way radio occultations by oblate axisymmetric atmospheres with zonal winds, 2015, Radio Science, 50, 712-727.

[2] A. Bourgoin, M. Zannoni, and P. Tortora: Analytical ray-tracing in planetary atmospheres, 2019, A&A, 624, A41.

[3] E. Poisson and C. M. Will: Gravity: Newtonian, Post-Newtonian, Relativistic, 2014, Cambridge University Press.

[4] M. Born and E. Wolf: Principles of Optics, 1999, Cambridge University Press.