

The multi-shell models of celestial bodies with an intermediate layer of fluid: dynamics in the case of the large values of the Ekman number

Vladislav Sidorenko (1), Sergey Ramodanov (2)

(1)Keldysh Institute of Applied Mathematics, Moscow, Russia, (2) Mechanical Engineering Research Institute, Moscow, Russia (vvsidorenko@list.ru)

Abstract

The structure of celestial bodies can often be described as a set of three components: an internal solid core, an intermediate liquid layer, and an outer solid shell. If the viscosity of the fluid is large enough (or, more precisely, the Ekman number $E \gg 1$), then the consideration of the rotational motion of multi-shell system can be reduced to studying the rotation of a rigid body under the action of a torques of a special form.

1. Prelude

In some studies on rotational dynamics of planets and satellites these celestial bodies are considered as a set of three components: an internal solid core, an intermediate liquid layer, and an outer solid shell. The contribution of these components to the total mass of the body is significantly different in objects of different nature. For example, for planets, the mass of the inner core is much less than the mass of the outer solid shell, called the mantle in planetary physics (Figure 1a). For some satellites of Jupiter and Saturn, the outer shell is an ice layer whose thickness is substantially less than the radius of these objects (Figure 1b). Depending on which component is dominant, the reference frame in the studies of rotational motion of such multi-shell models is connected either with the outer shell and then the relative motion of the liquid and inner core is considered, or with a massive inner component and then the relative displacements of the outer shell are investigated.

One of the key parameters in the theory of rotating fluids is the Ekman number [1]:

$$E = \frac{\mu}{\rho_f \omega_{rot} R_c^2},$$

where μ and ρ_f are the viscosity and the density of the liquid, ω_{rot} is the characteristic angular velocity, R_c is the outer radius of the cavity filled with liquid.

If $E \gg 1$, then the motion of the liquid in the cavity is characterized as creeping one.

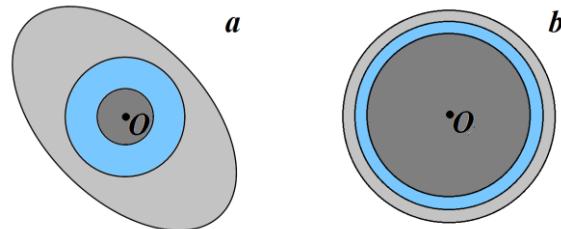


Figure 1. Examples of celestial bodies described by multi-shell models with an intermediate liquid layer: a – “planet” (small solid inner core), b – satellite with a thin outer ice shell.

2. Main results

We obtain approximate formulas describing the velocity field in the liquid layer in the case of creeping flow. Then, following [2,3], we show that under the condition $E \gg 1$ the consideration of the rotational motion of a multi-shell system can be reduced to studying the rotation of a “equivalent” rigid body under the action of a torque of a special form.

Acknowledgements

Our investigations were supported by RFBR (grant 17-01-00902).

References

- [1] Greenspan, H.P.: The theory of rotating fluids, Cambridge University Press , 1968.
- [2] Chernousko, F.L.: Motion of a rigid body with cavities containing a viscous fluid, NASA Technical Translations, 1972.
- [3] Ramodanov, S.M. and Sidorenko, V.V.: Dynamics of a rigid body with an ellipsoidal cavity filled with viscous fluid, International Journal of Non-Linear Mechanics, Vol. 95, pp. 42-46, 2017.