

# Stochastic charging as transport mechanism of Saturn’s ring rain

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## Abstract

Recent studies [7, 2, 6] have found, that there is a *ring rain* onto Saturn: Particles that originate in Saturn’s rings and ‘fall’ along magnetic field lines into the planet’s atmosphere. We study, whether a successive on- and off-hopping of charge carriers, i.e. quantised charging processes, can act as a transport mechanism to remove nano-particles from the rings and add them to the ring rain.

## 1. Introduction

Impacts of interplanetary micro-meteoroids erode the inner rings of Saturn and produce dust particles that get charged by the surrounding plasma environment [6]. Under certain circumstances, these particles can leave the ringplane, gyrating around the magnetic field lines, and approach Saturn where they are eventually absorbed by the planet’s atmosphere. O’Donoghue et al. [7, 2] already had described the  $H_3^+$  imprint of that ring rain, that can be seen by infrared observations, when Cassini’s CDA measured the particles directly during the final phase of the mission [6].

For nano particles that can only carry one or a few excess charges, the charging process cannot be described by a continuous change of the charge  $\dot{q} = \sum I_\alpha(q)$ , that is given by the sum of currents  $I_\alpha$ , e.g. the photoelectric current or electron collection (cf. supplementary materials of [6]). The charge is changed by isolated charging events, i.e. gain or loss of charge carriers, and jumps between discrete states. This repeated change of the charge dependent gyration frequency is capable of increasing the particle’s energy.

We use a stability measure coming from the field of nonlinear dynamics to calculate the average growth rate of the energy. This enables us to determine an average rate at which the particles travel from the rings to the planet.

## 2. Gyration ring particles as parametric oscillators

We look at the equations of motion of a charged particle under the influence of the gravity of Saturn (with oblateness) and the planetary magnetic field (using the Z3 model [1]). Performing an epicyclic expansion, we end up with oscillator equations

$$\ddot{x} = -\omega^2(t)x$$

for the radial and vertical component of the particle’s position in a planetocentric frame. The oscillator frequency  $\omega$  switches between two states  $\omega_{1/2} = \omega_0 \sqrt{1 + \xi_{1/2}}$ , with an average value  $\omega_0$ , in the most simple case. The transition times between those two states are exponentially distributed [5] with average rates  $\lambda_{1/2}$ . The values of  $\xi_{1/2}$  and the transition rates are determined by the system parameters (particle mass, magnetic and gravitational moments...) and the charging currents around the equilibrium charge.

We used a method described by Horsthemke and Lefever [4] to find the probability density  $P(\phi)$  for the phase-like variable  $\phi$ , defined by  $\cot \phi = \frac{\dot{x}}{x}$ .

## 3. The Lyapunov exponent of a parametric oscillator

The Lyapunov exponent is generally a measure of the stability of a system. It is defined as

$$\Lambda = \lim_{t \rightarrow \infty} \frac{1}{2t} \overline{\ln E}$$

with the energy-like variable  $E = \frac{\dot{x}^2}{2} + \frac{x^2}{2}$ . Mallick and Marcq [3] showed, that for an Ornstein-Uhlenbeck process  $\xi(t)$ , in the stationary limit, the Lyapunov exponent is given by  $\Lambda = \langle \cot \phi \rangle$ , where the brackets indicate averaging with respect to the stationary distribution  $P(\phi)$ . That this holds, even if the noise  $\xi(t)$  is dichotomic.

## 4. The extraction rate of dust particle exposed to subsequent discrete charging events

Even for parameter sets, that describe a stable particle trajectory, as long as no charging is considered - so sets that result in real values of  $\omega$ , we find  $\Lambda > 0$ , as soon as we apply dichotomic charging. Assuming the simple description  $\lim_{t \rightarrow \infty} \langle x \rangle(t) \propto e^{\Lambda t}$ ,  $\Lambda$  describes an extraction rate for the particles from the rings. We will present values of  $\Lambda$  for different sets of particle and plasma parameters.

## 5. Summary

Using stochastic methods, we found a way to give the time scales on which the particles wander along the field lines. This will allow us to determine, whether stochastic charging is an effective transport mechanism for the ring rain particles.

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