

# A comparison between physical and classical prescriptions of isostasy

Antony Trinh (1,2), Mikael Beuthe (1) and Isamu Matsuyama (2)

(1) Royal Observatory of Belgium, Belgium, (2) Lunar and Planetary Laboratory, University of Arizona, Arizona, USA  
(a.trinh@oma.be)

## Abstract

We compare classical, computational prescriptions of isostasy, with more physical prescriptions based on minimisation of stress or energy.

## 1. Introduction

Isostasy, a state of equilibrium where the crust floats over a fluid-like mantle or fluid ocean, is one of the possible mechanisms that can explain the persistence of the surface topography of planets and moons. On flat bodies, isostasy implies that columns of equal cross-sectional area should have equal mass above the depth of compensation, or equivalently, that the pressure excess entailed by the surface topography should cancel at the depth of compensation. Although these equal-mass and equal-pressure prescriptions can be extended to spherical geometry [3], they then become computational in nature, and inequivalent to one another because of curvature and self-gravitation. A more physical prescription consists in minimising the norm of the deviatoric stress tensor ([2] in local isostasy, [1] in regional isostasy), or the total (elastic + gravitational) energy.

## 2. Method

We here assume Airy compensation, where the surface topography reflects the topography at the bottom of the crust, and the protrusion of crustal material into the denser mantle or ocean results in ‘negative mass’ at depth that gravitationally compensates the surface topography. We consider a non-rotating, three-layer Enceladus with surface topography, and compute crustal stresses and gravity field for a range of bottom topographies by self-consistently solving the equations of static equilibrium in near-spherical geometry.

## 2.1. Local and regional isostasy

Balance of momentum leaves three components of the stress tensor unconstrained, and we must specify three additional constraints to close the problem. In local isostasy, the radial-tangential stresses are assumed to vanish, and for definiteness we assume an additional relation between the stress components, given by Eq. (19) in [2]. In regional isostasy, the stress tensor is assumed to be that of an incompressible isotropic elastic material, so that the norm of the deviatoric stress tensor is the lowest possible.

## 2.2. Isostatic prescription

We consider four different prescriptions of isostasy, corresponding to four different bottom topographies. The equal-mass and equal-pressure prescriptions impose a bottom topography given by Eqs (2) and (3) of [3], respectively, independent of the state of stress. The minimum-stress and minimum-energy prescriptions select the bottom topography for which the norm of the deviatoric stress tensor and the total energy, respectively, are minimised.

## 3. Results

Fig. 1 shows the predicted bottom topography for the various isostatic prescriptions. At the largest scales (lowest degrees), the local and regional minimum-stress prescriptions are equivalent; the equal-pressure prescription slightly underestimates the bottom topography to 95% of its regional minimum-stress value, while the equal-mass prescription overestimates the bottom topography by at least 20%. At smaller scales (higher degrees), the computational prescriptions largely overestimate the bottom topography, which is typical of local isostatic prescriptions due to the lack of lithospheric flexure corrections. The minimum-energy prescription results in virtually no compensation.

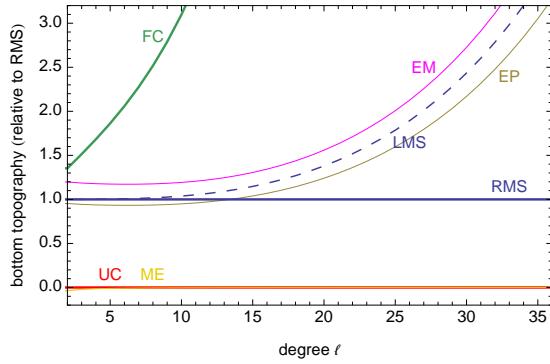


Figure 1: Bottom topography associated with various isostatic prescriptions (EM: equal-mass, EP: equal-pressure, LMS: local minimum-stress, RMS: regional minimum-stress, ME: minimum-energy) and various limits (UC: uncompensated, FC: fully compensated), relative to the regional minimum-stress (RMS) prescription.

Fig. 2 shows the norm of the deviatoric stress tensor for the various isostatic prescriptions. The stresses are indeed always smaller in regional isostasy than in local isostasy (the solid lines are always below the corresponding dashed lines). In addition, the regional minimum-stress prescription indeed realises the minimum stresses (no curve goes below 1). Interestingly, the fully compensated limit, generally considered as a sign of isostasy, is associated with large stresses.

Fig. 3 shows the gravity coefficient, or equivalently the admittance, for the various isostatic prescriptions. The equal-pressure prescription tends to overestimate the admittance by up to 15%, while the equal-mass prescription tends to underestimate the admittance to as little as 45% of its regional minimum-stress value.

## Acknowledgements

A. T. was supported by the European Research Council under Grant No. 670874 issued through the European Union’s Horizon 2020 research and innovation programme. M. B. is supported by the Belgian Science Policy Office under Grant No. 4000120791 issued through the PRODEX programme. I. M. and A. T. are supported by the National Aeronautics and Space Administration (NASA) under Grant No. 80NSSC17K0724 issued through the NASA Solar System Workings program.

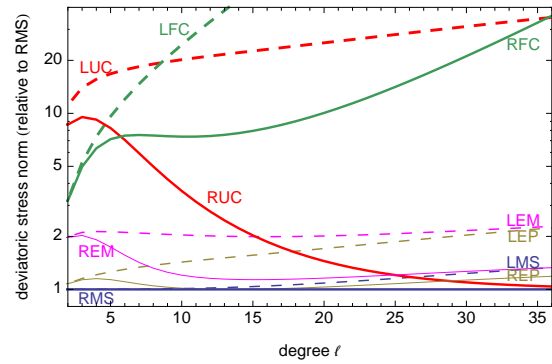


Figure 2: Norm of the deviatoric stress tensor associated with various isostatic prescriptions and various limits in local (prefix L) or regional (prefix R) isostasy, relative to the regional minimum-stress (RMS) prescription.

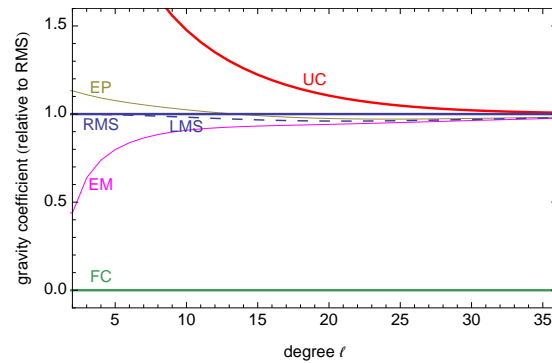


Figure 3: Gravity coefficient, or equivalently admittance, associated with various isostatic prescriptions and various limits, relative to the regional minimum-stress (RMS) prescription.

## References

- [1] Beuthe, M., Rivoldini, A., and Trinh, A.: Enceladus’s and Dione’s floating ice shells supported by minimum stress isostasy, *Geophysical Research Letters*, Vol. 43, pp. 10088-10096, 2016.
- [2] Dahlen, F. A.: Isostatic geoid anomalies on a sphere, *Journal of Geophysical Research: Solid Earth*, Vol. 87, pp. 3943-3947, 1982.
- [3] Hemingway, D. J. and Matsuyama, I.: Isostatic equilibrium in spherical coordinates and implications for crustal thickness on the Moon, Mars, Enceladus, and elsewhere, *Geophysical Research Letters*, Vol. 44, pp. 7695-7705, 2017.