EPSC Abstracts
Vol. 13, EPSC-DPS2019-1785-4, 2019
EPSC-DPS Joint Meeting 2019

EPSC
EPSC-DPS Joint Meeting 2019
15–20 September 2019 | Geneva, Switzerland

Asteroid and comet shape modeling using overcomplete spherical bases

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Abstract

Spherical harmonics form a complete basis on the sphere, i.e., they can present any squared-integrable function on the sphere given that the degree and order of the bases are enough. For this reason, it is convenient to represent asteroid or comet shapes using spherical harmonics [1]. Nevertheless, it suffers some drawbacks when it comes to modeling the multi-scale local details of the shape. When the local irregularity is large, spherical harmonics representation will become less efficient and require large amount of coefficients. In this work, we will present a method for modeling the shape of asteroids and comets using so-called overcomplete spherical bases. That is, we combine both spherical harmonics for a global representation and spherical wavelets for local detailed modeling. Since spherical wavelets and spherical harmonics are both compete bases on the sphere, they form overcomplete bases. This method will bring some advantages to both the representation and analysis of the shape of asteroids and comets. The analysis capability comes from that the coefficients of the bases could be rotationally invariant, and the representation advantage comes from the compression capability of spherical wavelets.

1. Motivation

In addition to composition, the shape of asteroids and comets is an important physical property. The knowledge of shape could provide insight on how they form in the first place , their evolution mechanism, internal structure information, and interaction with other objects. Additionally, for spacecraft operations around the asteroid or comet, a reliable and realistic shape model is needed. Furthermore, understanding the shape of these objects could also provide insight on how to detect them. Therefore, modeling the shape of these objects are important.

2. The overcomplete spherical bases

The overcomplete spherical bases (or redundant bases) are in contrast with a complete spherical basis set. It means that it is more than enough to represent or analyze a function using these bases. However, by using the combination of different families of basis functions, the representation, analysis of a complex function often become more flexible and efficient. For example, one can obtain a spherical wavelet representation of Gaussian random sphere by simply representing the spherical harmonics basis functions using the wavelet bases. In this work, we use three types of basis families, i.e., the spherical harmonics, discrete spherical wavelets, and the continuous spherical wavelets (also called overcomplete spherical wavelets).

2.1. Spherical Harmonics

Spherical harmonics are the most commonly applied basis functions on the sphere.They arise from searching the angular solution of Laplace's equation in spherical coordinates. It is a powerful analytical tool since it is the Fourier analogue on the sphere.

$$y_l^m(\theta,\phi) = \begin{cases} \sqrt{2}K_l^m cos(m\phi)P_l^m(cos(\theta)) \\ K_l^0 cos(m\phi)P_l^0(cos(\theta)) \\ \sqrt{2}K_l^m sin(-m\phi)P_l^{-m}(cos(\theta)) \end{cases}$$

2.2. Discrete Spherical Wavelets

Discrete spherical wavelets are defined on the hierarchical geodesic grids on the sphere [2]. These grids can be obtained by subdividing the zero-level grids, the icosahedron. The discretization of a spherical surface in such a way leads to the multi-scale representation of a function on the sphere. The bases for different scale representation are called the scaling functions

ϕ_k^j, and the spherical wavelets $\tilde{\psi}_m^j$ live in the complementing space between two nested scaling function spaces.

$$f = \sum_{k \in K^0} c_k^{J_0} \tilde{\phi}_k^{J_0} + \sum_{j=J_0}^{J} \sum_{m \in M^j} d_m^j \tilde{\psi}_m^j,$$

2.3. Continuous Spherical Wavelets

Continuous spherical wavelets differ from the discrete spherical wavelets in three ways. First, the continuous one can be defined on every possible point on the spherical surface. Second, the construction of the multi-scale wavelet basis is through the dilation and inverse-stereographic-projection of wavelets on a plane. Third, the multi-scale wavelet coefficients are obtained via spherical convolution (this operation can only be done on frequency-domain). Consequently, the multi-scale wavelet coefficients will be a function of rotation angle. More details are described in [3], [4], [5], [6]. Continuous spherical wavelets provide more power on shape analysis of the asteroids and comets. Owing to the fact that the coefficients are connected with rotation and scale, it could also provide some insight on detecting and imaging the rotating bodies.

3. The construction, analysis, and representation of the shape of asteroids and comets

In this work, we define the term "modeling" in a more specific way, i.e., it is a composition of construction, analysis, and representation of the asteroids and comets shape. According to different features of these basis families, we devise our modeling scheme as displayed in Figure 1.

4. Summary and Conclusions

We present a novel method for modeling the shape of asteroids and comets using overcomplete spherical bases. The method could potentially benefit the study of the shapes in terms of their construction, analysis, and representation.

Acknowledgements

This work was supported by the Academy of Finland project No. 298137 ALBEDO.

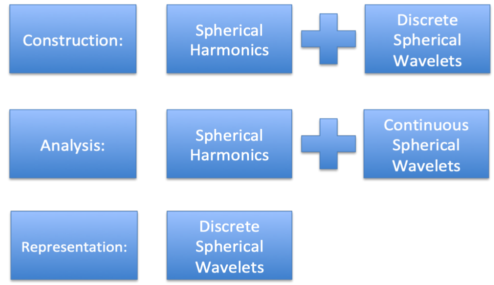

Figure 1: Asteroid and comet shape modeling using overcomplete spherical bases

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