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Eros disruption by Earth at near flyby: possible scenarii

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Abstract

We discuss the possible breakup of a prolate asteroid during a close flyby. New constraints for the tidal disruption for solid biaxial ellipsoids are derived, taking the size, shape, density, material strength, compressibility, rotation, and orientation of the flyby object into account using an exact analytical solution. Results set a lower limit for the critical distance for a break-up to occur, relative to the rubble-pile models, in relation to reasonable values of tensile strength and a Poisson ratio. Exact expressions for the stress tensor components under self-gravity, rotation and tidal forces are derived using methods from elasticity theory. Asteroid 433 Eros at flyby near Earth is presented as a case study.

1. Introduction

One of the most important questions is at what distance from a large planet a small body may split, and how this distance depends on the physical properties and figure of the body. This has been under debate ever since Edouard Roche (1864), in the middle nineteenth century, drew attention to the problem by calculating his famous expression for the splitting distance, D, as

$D = 2.45\ (\rho_p/\rho_0)^{1/3} R_p$ or $\delta = D/R_p\ (\rho_0/\rho_p)^{1/3} = 2.45$, (1)

where R_p and ρ_p are the radius and density of the planet, and ρ_0 is the density of the small body. Roche used a homogeneous, self-gravitating liquid satellite on a circular orbit around a solid planet, rotating in a bound, direct fashion along an axis perpendicular to the orbital plane. He only took the effects of self-gravity and tidal forces into account; the resulting critical breakup distance showed no reference to any other property of the body than its density. Later, ellipsoidal figures of solid bodies under tidal and self-gravity forces were studied [1-3]. But, a number of constraints, like incompressible body etc., not allowed getting a general solution.

2. Analytical procedure

The well-understood theory of gravitational and tidal potential can be used to calculate elastic deformations and stresses of small bodies. The force field strength for a solid body **F** is determined by the gradient of the total potential

$\mathbf{F} = \rho_0\, \mathrm{grad}[V(\mathbf{r})],\ V(r) = V_g(\mathbf{r}) + V_\omega(\mathbf{r}) + V_t(\mathbf{r})$ (2)

where ρ_0 is the mean density, V_g is the gravitational potential, V_ω is the centrifugal potential, and V_t is the tidal potential. If the small body has an axis of symmetry, the problem simplifies to an exact analytical solution. In general, the Cartesian decomposition is appropriate, as in

$$\mathbf{u} = \mathbf{e_x}\, u_x + \mathbf{e_y}\, u_y + \mathbf{e_z}\, u_z$$

Then, the equilibrium equation of an isotropic body in the total (gravitational, tidal and centrifugal) force field takes the form

$$\mu\, \Delta\mathbf{u} + (\lambda + \mu)\ \mathrm{grad}\ (\mathrm{div}\ \mathbf{u}) = -\mathbf{F}, \quad (3)$$

where μ, λ are the Lamé constants and

$$\mu = E/2(1+\nu);\ \lambda = \nu E/(1+\nu)(1-2\nu), \quad (4)$$

where ν is the Poisson ratio, and E is the Young modulus of solid bodies. If the surface is free of load; as it is usually the case for small bodies and asteroids, then $f_i = 0$ on the surface as boundary conditions.

The total stress in the body is determined by the composition of tensile (centrifugal and tidal) and compressive (gravity) forces. Their values depend in turn on the intensity ratios of the respective potentials

$$I_\omega = \pi/T^2 G\rho_0,\ \omega = 2\pi/T;$$
$$I_t = MD^3/2\pi\rho_0\ = 2/3\ 1/\delta^3, \quad (5)$$

where G is the gravitational constant , T is the period of rotation, M is the mass of the planet, D is the distance between the centers of the planet and the small body. Unfortunately, the elegant method of decomposition on spherical harmonics, is not applicable for asteroids at considerable non-sphericity of a figure. An elongated small body is better represented by a biaxial ellipsoid with the main semiaxes a and c, a > c, with the eccentricity

$\varepsilon = \sqrt{(a2 - c2)}/a$

For example, the figure of Eros is well approximated by an elongated biaxial ellipsoid with $\varepsilon \approx 0.946$.

The tidal potential, $V_t(x,y,z)$, in a general flyby-scenario is described by the following expression

$V_t(\mathbf{r}) = - G*M/D^3 [\mathbf{r} - 3*\mathbf{R}*(\mathbf{R}*\mathbf{r})/R^2]$,

where **r** is (x,y,z) in the local coordinate system associated with the principal axes of the body, and **R** is (D,θ,φ) in the corresponding spherical coordinate system, pointing toward the flyover planet. In special cases, $\theta = \pi/2$ and $\varphi = 0$ or $\varphi = \pi/2$, and the tidal potential could be represented by simple quadratic forms:

$V_t(x,y,z) = G*M/D^3 (x^2 - y^2/2 - z^2/2)$,

$\varphi = 0$, "down" case; (6)

$V_t(x,y,z) = G*M/D^3 (y^2 - x^2/2 - z^2/2)$,

$\varphi = \pi/2$, "flat" case.

In these special scenarios, the long axis points to the flyover planet ("down" case) or is perpendicular to it ("flat" case).

3. Results

The final solution for the diagonal components of the stress tensor is convenient to present in a normalized form

$\sigma_{11}(x,y,z) = \sigma_0 [S^{(1)}_{11}(\nu,\varepsilon,I_\omega,I_t) ((x/a)^2 - 1) + S^{(2)}_{11}(\nu,\varepsilon,I_\omega,I_t) (y/c)^2 + S^{(3)}_{11}(\nu,\varepsilon,I_\omega,I_t) (z/c)^2]$ etc.

where $\sigma_0 = 4\pi G(\rho_0 a)^2$. This representation explicitly expresses the boundary condition of the free surface and allows us to estimate the spatial distribution of stresses in the asteroids body for different cases of its orientation. Everywhere further we will use the asteroid Eros as an example.

Thus, for Eros, the tensile (positive) stress arises from a certain distance along the axis directed at the planet: for the orientation "down" is the axis OX (see Fig. 1a), for the orientation "flat" is the OY axis (see Fig. 1b). And for the last case, this distance is significantly less. With moderate rotation, along other axes of the body stress is compressive, only.

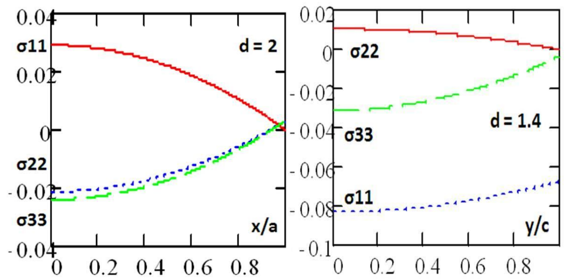

Figure 1: Stress components (at σ_0) (a) along OX case «down»; (b) along OY case "flat", $d=D/R_E$

4. Summary and Conclusions

Thus, only for the orientation "down" the possibility of complete decay of a prolate small body under tidal forces when approaching the planet is realized. A necessary condition for this is the presence of tensile stresses on some cross-section. Previous analysis showed that for the lower estimate of the critical distance Dc to the planet it is best to use the value σ11 in the center of the body (cross-section x=0) as $\sigma^{(c)}_{11}$ (D, T, ν, ε) = 0.

So, $\delta_c = \delta(\nu,\varepsilon)$ will depends on ν and ε of the body.

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