

## On the asphericity of the figures of Pluto and Charon. Triaxial approximation

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### Abstract

The NASA spacecraft *New Horizons* allowed to refine many physical, chemical, topographic, and geological characteristics of main bodies of the Pluto – Charon system significantly [1, 2, 3]. Their sizes have been clarified. But asphericity of the figures was not revealed (measurement errors of the polar  $\alpha$ , and equatorial  $\beta$  oblateness exceed their values). The same concerns the non-sphericity of the gravitational field: the measurement errors of the second zonal  $I$  and sectorial  $J$  harmonics exceed their values. It seems relevant to find their values theoretically under various assumptions about the structure of the bodies. Here we have obtained the bilateral boundaries of the possible values of  $\alpha$ ,  $\beta$ ,  $I$ , and  $J$  for Pluto and Charon under the assumption of hydrostatic equilibrium of both bodies. It turned out that centrifugal and tidal forces are comparable. We have found that the values of above quantities have the order of  $10^{-5} \div 10^{-4}$  for Pluto, and  $10^{-4} \div 10^{-3}$  for Charon.

### 1. Introduction

We consider an isolated two-body system  $T_1$  (Pluto) and  $T_2$  (Charon). We assume that the body  $T_i$  is bounded by triaxial ellipsoid  $S_i$  with semiaxes  $a_i > b_i > c_i$ , and the whole system rotates as a solid with the angular velocity  $\omega$  around the fixed axis  $z$ . Let us introduce oblatenesses  $\alpha_i = (a_i - c_i)/a_i$ ,  $\beta_i = (a_i - b_i)/a_i$  of sections  $S_i$  by planes  $x_i y_i$  and  $x_i z_i$ , respectively. As usual,  $c_i$  denotes the polar semiaxis,  $a_i$  denotes the semiaxis directed toward the body  $T_{i\pm 1}$ . We assume the flattenings  $\alpha_i, \beta_i$  being small first-order quantities, and neglect quantities of the second order.

We regard the ellipsoid  $S_i$  as a level surface of sums of gravitational potentials  $V_i$  and centrifugal potential

$W$ . Thus, if the point  $Q$  belongs to  $S_1$ , then

$$V_1(Q) + V_2(Q) + W(Q) = \text{const.} \quad (1)$$

Relation (1) is valid for  $Q \in S_2$  also, but with another constant at the right hand side. In spherical coordinates (1) takes the form

$$V_1 + V_2 + W = B_1 + B_2 \cos^2 \theta + B_3 \sin \theta \cos \lambda + B_4 \sin^2 \theta \sin^2 \lambda + \dots \quad (2)$$

Here  $B_k$  are simple functions of  $\alpha_1, \beta_1, I_1, J_1$ , and known from observations parameters; the truncation term has the second order of smallness.

### 2. Results

The condition (1) implies  $B_2 = B_3 = B_4 = 0$ . Solving these equations, we obtain relations between oblateness and Stokes coefficients

$$I = -\frac{1}{3}\beta + \frac{2}{3}\alpha - \sigma_1, \quad (3)$$

$$J = \frac{1}{6}\beta - \sigma_2,$$

$$\sigma_1 = 9.686 \cdot 10^{-5}, \quad \sigma_2 = 6.782 \cdot 10^{-6}.$$

Similar equations we obtain for Charon.

As it is known, minimal possible values of Stokes coefficients are  $I = J = 0$ , which leads to minimal possible values of flattenings. Maximal flattenings have the homogeneous ellipsoid. The Stokes coefficients of the last figure are well known. Hence, we can easily find maximal possible values of Stokes parameters, and flattenings. The bilateral estimates of these quantities are gathered in Tables 1, and 2 in clear notations.

### 3. Summary and Conclusions

We find bilateral estimates for polar and equatorial oblateness  $\alpha$ ,  $\beta$  of Pluto and Charon, as well as bilateral estimates of their Stokes coefficients  $I$ ,  $J$  characterizing the second zonal and sectorial harmonics. Numerical results are gathered in two tables. They show us, how precise must be measurements capable to obtain these quantities from observations with an accuracy of 10% at least.

### 4. Tables

Table 1: Boundaries of the oblateness values  $\alpha$ ,  $\beta$

Body	$10^4\alpha^-$	$10^4\alpha^+$	$10^4\beta^-$	$10^4\beta^+$
Pluto	1.7	4.1	0.4	1.0
Charon	5.0	12.5	3.6	9.1

Table 2: Boundaries of the Stokes parameters  $I$ ,  $J$

Body	$I^-$	$10^4I^+$	$J^-$	$10^4J^+$
Pluto	0	1.5	0	0.10
Charon	0	3.2	0	0.91

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