

# Material Mixing in the Earth-Moon Formation Scenario

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## Abstract

In this work, we discuss the results of a survey of collision simulations using an adaptive smoothed particle hydrodynamics code (SPHERAL), comparing many of the most commonly used SPH methods as well as a higher-order, reproducing kernel method (RPKM) that better resolves material mixing. These efforts are aimed at resolving the underlying causes of the discrepant results of Moon-forming collision simulations in the literature, as well as advocating for a common methodological framework for studies of this kind, based on hydrodynamical simulations that isolate the various physical processes at work. We also discuss the central roles that initial conditions for the Earth and impactor and the choice of artificial viscosity play in the subsequent mixing events of the constituent layers of both bodies.

## 1. Introduction

The giant impact hypothesis for the formation of Earth's moon remains a subject of great interest and study, with many unanswered questions about its feasibility for reproducing the Earth-Moon system as we observe it today [1][2]. One of the most important outstanding problems is that of fashioning a collision scenario that simultaneously reproduces the relatively high angular momentum of the Earth-Moon system, as well as the consummate mixing of proto-Earth and impactor material such as to produce the apparent similarity of isotope abundances in Earth and Moon rocks. High angular momentum implies a high-velocity (and/or) grazing collision [3], whereas the inability to detect significant isotopic differences between Earth rocks and Moon rocks returned by Apollo astronauts implies a collision scenario where material from both bodies was evenly mixed.

To date, many of the disparate conclusions about the necessary physical processes required to produce this result can be tied to pernicious numerical artifacts of the SPH method. Spurious surface tension at material interfaces may be responsible for insufficient dredge-up of proto-Earth core material, the well-known "E0" error of SPH may be responsible for insufficient mixing of the proto-Earth and impactor mantle material, and overly diffusive, intermittently activating artificial viscosity prescriptions may incorrectly transport angular momentum to large radii in the collisionally formed disk, resulting in too much material falling back onto the proto-Earth. Here, we examine the effects of these numerical artifacts on a collection of Moon-forming SPH simulations in order to quantify the extent to which each of these artifacts might erroneously inform conclusions about the Moon's formation.

## 2. Methods

For our simulations, we employ an adaptive smoothed particle hydrodynamics code (SPHERAL), developed at Lawrence Livermore National Laboratory (LLNL). We use the LEOS equation of state and the RPRPS generator [4] to ensure conformal particle distributions. We also use a Lane-Emden type differential solver to solve for the steady-state, low-energy density and pressure profiles for two-component planetoids with iron cores and rocky mantles. Previous work [4] has demonstrated that initial conditions of this type are best able to approximate the steady-state condition of the present-day Earth in SPH simulations, and so are well suited to this problem.

For a number of our simulations, we compare the traditional density-energy hydro-solver of standard SPH, the density-independent SPH method (DISPH) [5], and the reproducing kernel method of CRKSPH [6]. We also evaluate the effect of different prescriptions for artificial viscosity (and AV limiters like the Balsara switch) on the relative abundances of target and impactor material in the collisionally formed proto-Lunar disks.

## 3. Figures

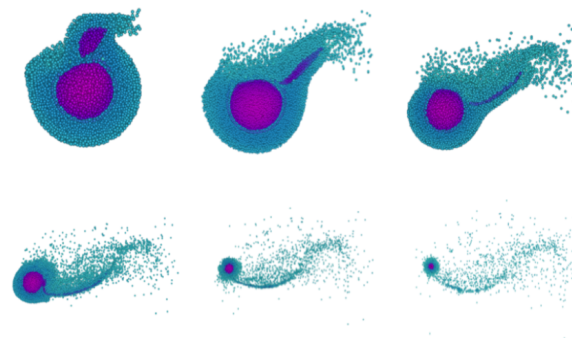


Figure 1: A simulation of a grazing collision of a proto-Earth and a Mars-sized impactor in standard SPH with a linearly-limited artificial viscosity depicting 5.5 hours of simulation time. Iron is indicated with purple, while basaltic material is cyan.

## 4. Summary and Conclusions

Many of the seemingly contradictory results of prior Moon-formation simulations can be tied to numerical deficiencies of the standard SPH method. Oftentimes, insufficient mixing is the result of SPH's E0-error, and Lunar formation disks that seem

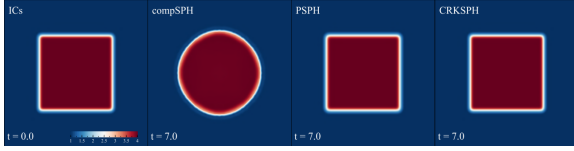


Figure 2: The initial and final conditions ( $t = 0$ ,  $t = 7$ ) of the hydrostatic box test at resolution  $N = 100^2$ , for compSPH, PSPH (DISPH), and CRKSPH, respectively. Artificial “surface-tension” errors at the contact discontinuity, incurred from assuming continuity in the material density, cause classical SPH to deform the square into a circle. Both PSPH and CRKSPH maintain the equilibrium. PSPH avoids the tension error by discretizing in pressure (a smooth quantity in this problem), whereas CRKSPH uses corrected kernels to accurately interpolate the interface.

to abruptly collapse are often plagued by overly diffusive artificial viscosity. While these numerical deficiencies are serious enough to drastically alter the evolution of any simulation, leading to unphysical results, they are nevertheless surmountable problems that can be eliminated through careful examination of each physical process in isolation, illuminating those processes where the assumptions that underlie the standard SPH method do not hold. When these numerical deficiencies are overcome, we may establish a common methodological framework for studying the Moon’s formation, and that framework must necessarily include some form of a higher-order kernel and a linearly-limited artificial viscosity.

## Acknowledgements

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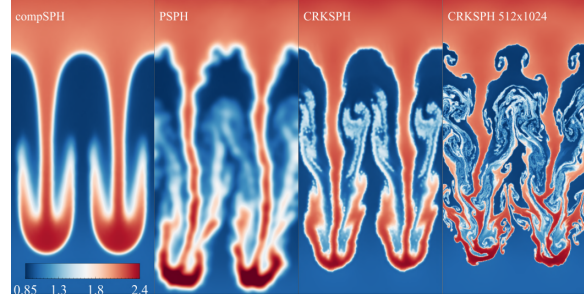


Figure 3: Pseudocolor plots of the density in Rayleigh-Taylor instability simulations. From left-to-right: compSPH with Monaghan-Gingold viscosity, PSPH (DISPH), and CRKSPH, all at  $128 \times 256$  particle resolution at a time  $t = 4$ . For reference, we include a high-resolution run of  $512 \times 1024$  using CRKSPH (far right). CompSPH methods performs reasonably, primarily due to our resolution choice of 4 radial neighbors, as well as a quintic kernel to avoid pairing instabilities. PSPH shows the greatest plunge depth and more perturbations growing along the rising bubble interfaces. CRKSPH maintains the sharpest interfaces of all the methods and is able to resolve secondary instabilities in the trailing plumes, all qualitative indications of less unwanted viscosity activation and noise control.

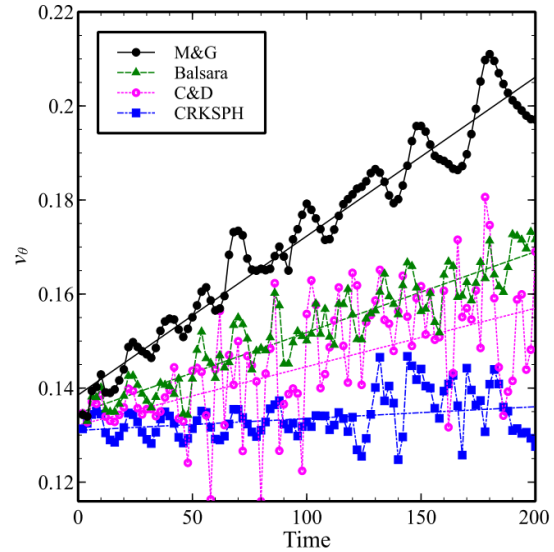


Figure 4: The evolution over time of the azimuthal velocity of a single particle in a generalized, static, rotating disk simulation for three common choices for artificial viscosity in SPH, plus the result of the higher-order accuracy CRKSPH simulation with its linearly-limited artificial viscosity. Any deviation from a zero slope in this plot is indicative of an unphysical, numerical acceleration of disk material as it gains angular momentum from material at lower radii, mediated by an overly aggressive artificial viscosity prescription.