

Stability around an equilibrium point of the comet 9P/Tempel 1

Gabriel Borderes-Motta (1), Tamires Moura (2) and Othon Cabo Winter(2)

(1) Bioengineering and Aerospace Engineering Department, Universidad Carlos III de Madrid, Leganés, Spain
(gabriel.borderes@uc3m.es)

(2) Grupo de Dinâmica Orbital e Planetologia, São Paulo State University, São Paulo, Brazil

1. Introduction

Nowadays, a large part of the missions in activity are directed to small bodies, i. e., missions whose targets are comets and asteroids. The mission OSIRIS-Rex from NASA [5] and the mission Hayabusa-2 from JAXA [4] are eminent examples, they are investigating the asteroids (101955) Benu and 162173 Ryugu, respectively.

As well as the number of missions, the works about the dynamics around small bodies grew up in last decades. The studies about this dynamics are useful to plan missions to irregular bodies and also identify features like rings, moons or debris.

In view of this we studied the dynamics around an equilibrium point of the comet 9P/Tempel 1 using the technique of Poincaré surface of section. A linearly stable equilibrium point is a dynamic characteristic which can indicate stable configurations of orbits around the body, mainly orbits that are synchronous with the spin movement of the body.

In next section we will provide some information on the tools used in this study, in the third section we will introduce some of the results, and in the forth section are some comments about implications of the results.

2. Model

The model used here is based in [1] were the dynamics around the asteroid 4179 Toutatis was mapped by Poincaré surface of section.

We use the polyhedron shape model of the comet 9P/Tempel 1 [3] to create a mascon model with 47722 point of mass whose sum of the masses is 7.2×10^{13} kg, i. e., the total mass of the comet. This model allows us to simulate the gravitational potential of this irregularly shaped body, and obtain accurate orbits in an efficient way.

The movement equations for this system are given by:

$$\ddot{x} - 2\omega\dot{y} = \omega^2 x + \frac{\partial}{\partial x} \sum_{i=0}^N \frac{Gm_i}{r_i} \quad (1)$$

$$\ddot{y} + 2\omega\dot{x} = \omega^2 y + \frac{\partial}{\partial y} \sum_{i=0}^N \frac{Gm_i}{r_i} \quad (2)$$

$$\ddot{z} = \frac{\partial}{\partial z} \sum_{i=0}^N \frac{Gm_i}{r_i} \quad (3)$$

where x , y and z are the coordinates of the system, ω is the spin velocity of the comet, G is the universal gravitational constant, m_i is the mass of each point of the mascons grid, N is the total number of points from the mascons grid, and r_i is the relative position of each mascon to the position of the orbiting particle.

We use the Burlish-Stoer integrator[2] to integrate the trajectories of particle under the same Jacobi constant. Then, we define a section that holds a linearly stable equilibrium point of the comet. Every instant of time when the trajectory crosses the section defined, we save the data of position and velocity in the x axis. The plot of this saved data gives us the Poincaré surface of section.

Note this construction of Poincaré surface of section is usually used to bi-dimensional problems. Even so, it is possible to take information by this tool with assistance of analysis of the third dimension, as it is shown in [1].

3. Results

Using the zero-velocity curves we identify 4 equilibrium points, where two points were identified as linearly stable equilibrium points. We chose the stable equilibrium point at the position $x = -1.93331$ km and $y = 13.5931$ km and we defined the cross section holding this point.

The Poincaré surface of section for the Jacobi constant 1.04×10^{-6} and the behavior in the third dimension are presented in the Fig. 1. The closed curves are

quasi-periodic orbits, a center common to a number of closed curves indicates a periodic orbit of first (low eccentricity) or second (resonant) kind. In Fig. 1 the closed curves are doubled. Each initial condition creates a pair of closed curves, one to the left and one to the right of the equilibrium point (at $x \sim 13.5$). It means that the movement of these quasi-periodic orbits and the periodic orbit is around the equilibrium point. The largest quasi-periodic orbit and the periodic orbit are in orange and purple, respectively, and its trajectories are shown in the same colors in Fig. 2, where it is possible to confirm they are orbiting the equilibrium point in the rotating frame.

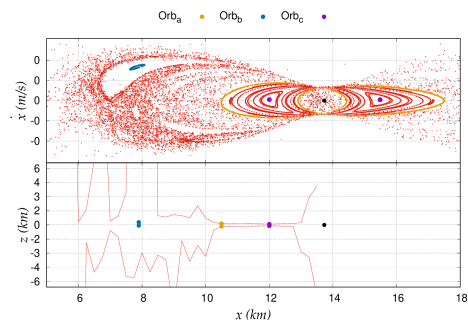


Figure 1: The Poincaré surface of section for the comet 9P/Tempel 1 with Jacobi constant 1.04×10^{-6} . Three conditions are marked in different colors blue, orange, and purple. The initial conditions have $\dot{y} > 0$ and $3.0 < x < 13.5$ km. The black dot is the linearly stable equilibrium point.

On the other hand, the randomly scattered dots can indicate chaotic trajectories, but due to the third dimension some trajectories creates scattered dots despite being periodic or quasi-periodic orbits. In order to discern if scattered dots indicate periodic or chaotic trajectory we can use the variation in the third dimension, i. e., if the variation in the third dimension is very lower than the variation of the other initial conditions that also generated scattered dots. To illustrate this, we selected an initial condition that generates scattered dots in blue and presents a low variation in the third dimension (view Fig. 1). The trajectory of this initial condition, showed also in blue in Fig. 2, is typically of a quasi-periodic orbit in resonance.

4. Final Comments

The results in the present works show the existence of stable regions in the vicinity of the equilibrium point. We intent to add the radiation solar pressure and ex-

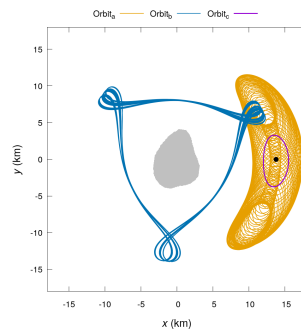


Figure 2: The orbits of the three selected initial conditions. In gray is the comet 9P/Tempel 1, in blue the orbit associated to a resonance, in orange a quasi periodic orbit, and in purple to a periodic orbit. The black dot is the linearly stable equilibrium point.

pand the study. The future goal is to check if the dynamics allows the comet hold particles during its inactivity period.

Acknowledgements

This work was financed in part by Coordenação de Aperfeiçoamento de Pessoal de Nível Superior - CAPES (Finance Code 001), CNPq (Procs. 305210/2018-1) and FAPESP (Procs. 2016/24561-0).

References

- [1] Borderes-Motta, G., and Winter, O. C., Poincaré surfaces of section around a 3D irregular body: the case of asteroid 4179 Toutatis, MNRAS, Vol. 474, pp. 2452-2466, 2018.
- [2] Bulirsch, R., and Stoer, J., Numerical Treatment of Ordinary Differential Equations by Extrapolation Methods, Jour. Numer. Math., Vol. 8, pp. 1-13, 1966.
- [3] Farnham, T. L., and Thomas, P. C., Plate Shape Model of Comet 9P/Tempel 1, V2.0, DIF-C-HRIV/TTS/MRI-5-TEMPEL1-SHAPE-MODEL-V2.0. NASA Planetary Data System, 2013.
- [4] Kawaguchi, J., Fujiwara, A., and Uesugi, T., Hayabusa-Its technology and science accomplishment summary and Hayabusa-2, Act. Astro., Vol. 62, pp. 639-647, 2008.
- [5] Lauretta, D.S. et. al., OSIRIS-REx: Sample Return from Asteroid (101955) Bennu, Spac. Sci. Rev., Vol. 212, pp. 925-984, 2017.