

A method for direct testing of hydrostatic equilibrium in exoplanet interiors

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Abstract

The characterization of the interior of exoplanets will unveil precious information on their formation, structure and evolution. The Love numbers h_2 and k_2 , which describe the planet's response to external perturbations, contain information on the radial density distribution and other interior parameters (e.g. viscosity). In the case of hydrostatic equilibrium, we simply have $h_2 = 1 + k_2$. We summarize how one can measure h_2 from transit observations and k_2 from periastron precession in eccentric orbits, hence providing a direct test for hydrostatic equilibrium in exoplanet interiors.

1. Introduction

It is widely assumed that hot Jupiters are in hydrostatic equilibrium [1], but this has never been verified by observations. If that is not the case, then information on orbital evolution and interior dissipation may be derived. Ultra-short period planets orbiting close to their Roche limit are subjected to strong tidal forces. The resulting radial displacement of the planet surface is proportional to h_2 , while the resulting redistribution of mass within the planet is proportional to k_2 [2]. In the case of hydrostatic equilibrium, both numbers are simply related through equation (1).

$$h_2 = 1 + k_2 \quad (1)$$

Radial displacements result in non-spherical transiting planets [3], and mass redistribution leads to an additional periastron precession term in eccentric orbits [4]. We summarize the adequate models to measure the Love numbers (Sections 2), and we give some hints on the target selection (Section 3).

2. Measuring the Love numbers

2.1 Measuring k_2

In eccentric orbits, the rotation of the semi-major axis (periastron precession) will occur because of general relativistic effects and because of tidal interactions in close in orbits. The rate of the latter contribution, ω_N is given by equation (2) [4, 6]

$$\begin{aligned} \omega_N = & \frac{nk_2}{2} \left(\frac{R_p}{a} \right)^5 \frac{F_p^2}{(1-e^2)^2} (1+q) + \frac{nk_2}{2} \left(\frac{R_p}{a} \right)^5 15q \frac{1+\frac{3}{2}e^2+\frac{1}{8}e^2}{(1-e^2)^5} + \\ & \frac{nk_{2,s}}{2} \left(\frac{R_s}{a} \right)^5 \frac{F_s^2}{(1-e^2)^2} \left(1 + \frac{1}{q} \right) + \frac{nk_{2,s}}{2} \left(\frac{R_s}{a} \right)^5 \frac{15}{q} \frac{1+\frac{3}{2}e^2+\frac{1}{8}e^2}{(1-e^2)^5} \end{aligned} \quad (2)$$

n is the mean motion, R_p is the planetary mean radius, R_s is the stellar mean radius, a is the semi-major axis, F_p and F_s are the ratio between the orbital and planetary (p) or stellar (s) rotations, respectively. q is the ratio between the stellar and planetary mass, e is the eccentricity, and $k_{2,s}$ is the stellar Love number. The latter can be obtained from theoretical calculations [4].

Periastron precession will induce transit timing variations (TTVs) and radial velocity variations (RVVs). For instance, the k_2 of an exoplanet has been measured for the time in WASP-18Ab, from TTVs and RVVs [6].

2.2 Measuring h_2

Surface deformations induced by tidal and rotational forces will modify the shape of an exoplanet from spherical to more complicated ones. The radius at any surface point of colatitude (θ) and longitude (ϕ) is given by Equation (3) [2, 7, 8].

$$R_p(\theta, \phi) = R_p \left(1 + q \sum_{j=2}^4 h_j P_j(\lambda) \left(\frac{R_p}{d} \right)^{j+1} - \frac{1}{3} h_2 (1+q) R_p^2 \left(\frac{R_p}{d} \right)^3 P_2(\cos \Theta) \right) \quad (3)$$

P_j is the Legendre polynomial of degree j , Θ is the obliquity, λ is a geometrical factor, and all other notations are the same as in Equation (2).

Non-spherical transiting planets exhibit a different transit shape compared to a transiting sphere. The detectability of k_2 in transit light curves has been emphasized using the above model [8], or using a simplified three-axis ellipsoidal shape model [9]. Both models require low noise levels (typically below 100-50 ppm/min) and a high precision in the stellar limb darkening.

3. Target selection

The higher the surface deformations, the easier the detectability of h_2 . Hence, planets orbiting within 3 Roche radii are the best for the retrieval of h_2 . On the other hand, planets in highly eccentric orbits with the periastron being around the transit time are also good targets.

The higher the eccentricity and the smaller the semi-major axis, the easier the detectability of k_2 .

Therefore, a good target will orbit close to its host star, the latter being massive enough to induce large surface deformations. The orbit will be eccentric, and if highly eccentric, the periastron should be around the transit time. For instance, the objects WASP-19b, HD80606b, and HAT-P-32b are good targets for independent measurements of h_2 and k_2 .

4. Summary and Conclusions

We propose a method for direct testing of hydrostatic equilibrium in exoplanet interiors. This method is based on independent measurements of the Love numbers h_2 and k_2 . The former can be measured from transits while the latter can be measured from periastron precession in eccentric orbits. If the condition $h_2 = 1 + k_2$ is not verified, then hydrostatic equilibrium is not reached. In addition, the independent measurements of the Love numbers put precious constraints on the planetary interior structure and orbital evolution.

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