

## Tides in a non-uniform subsurface ocean

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### Abstract

Tidal dissipation in subsurface oceans might play a relevant role in the evolution of icy moons. Different approaches have been used to study tides in subsurface oceans, however, the effect of topography has not been yet studied. We develop a new tool to study the barotropic tide in subsurface oceans using the commercial finite element software COMSOL Multiphysics. This new tool can be used to study ocean tides in subsurface oceans of changing topography. We use this code to study ocean tides in Enceladus. We consider an ocean with meridional topography covered by an ice-shell of uniform thickness and compute the tidal response for different ocean depths and ice shell thicknesses. We find that topography shifts the ocean resonant peaks.

### 1. Introduction

Tidal dissipation is an important contributor to the thermal budget of icy worlds with subsurface oceans (e.g., Europa, Enceladus, Triton). The non-zero eccentricity/obliquity of these moons results in a time-changing tidal potential which deforms the solid layers of these moons (ice-shell and rocky core) and results in heat production. Additionally, the tidal potential can drive ocean currents. Recently, it has been recognized that tidal dissipation within the oceans can also play an important role in preventing subsurface oceans from freezing. Two different approaches have been used, the shallow [1] and thick [2] ocean approaches. Here we expand on the shallow ocean approach and include the effect of variable ocean thickness to compute ocean tides.

### 2. Methodology

We use the linearised Laplace Tidal equations. We adopt the approach of [3] and [4] and use the Helmholtz decomposition, however, we keep the terms

associated with ocean thickness changes. We model the ice shell as a membrane of uniform thickness as in [5]. We do not include the effects of self-gravitation. We use a commercial finite element code, COMSOL Multiphysics, to solve the resulting equations. The problem is discretised on the surface of a sphere using triangular elements. The model is forced with the obliquity or eccentricity component of the tidal potential and run-forward in time using the Generalized-Alpha method until a steady-state is achieved. We benchmark our model using the analytical solutions of [6]. We find that as we increase the number of elements, our solution converges to that of [6].

Similarly to [7, 8] we use our model to study tidal dissipation for an ocean of constant thickness covered by an uniform ice shell. We then consider an Enceladan ocean with changing thickness. Enceladus' ocean topography is dominated by degree two and three zonal components [9, 10]. We consider the effect of the degree two topography to tidal dissipation, we use an ocean thickness given by:

$$h_{ocean} = \hat{h}_{ocean}(1 + n_{20}P_{20}), \quad (1)$$

where  $\hat{h}_{ocean}$  is the average ocean thickness,  $P_{20}$  is the degree two, order zero Legendre polynomial and  $n_{20}$  is a free parameter.

### 3. Results and Conclusions

As in [7, 8] we observe that the ice shell changes the ocean response to tides (Figure 1). We observe that for the eccentricity tide an ice shell modifies the resonance peaks and dampens the ocean response for thick oceans. Similarly, the obliquity resonance peaks are also shifted but the ice shell does not alter Rossby-Haurwitz resonance excited for thick oceans [1].

When we consider an ocean with degree two topography (Figure 2), we observe that the resonant peaks shift towards thicker oceans. However, we still find that tidal dissipation in oceans with realistic thickness is orders of magnitude smaller than the measured heat

flux. Moreover, we find that the topography hampers the Rossby-Haurwitz resonance.

The new tool can be used to study different more complex ocean geometries for other objects with subsurface oceans (e.g., icy moons, exoplanets)

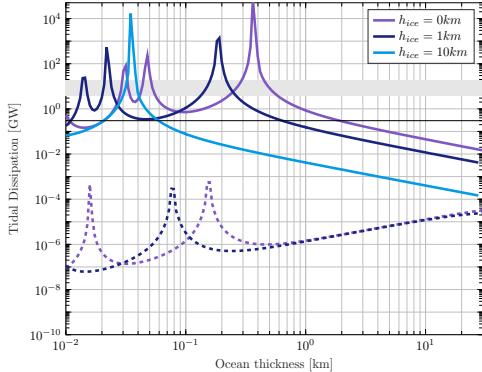


Figure 1: Tidal dissipation in an Enceladan subsurface ocean due to the eccentricity tide (solid lines) and obliquity tide (dashed lines) for an different ocean depths and ice shell thickness. A Rayleigh friction coefficient of  $\alpha = 10^{-7} \text{ s}^{-1}$  has been used. The shaded region indicates measurements of Enceladus heat flux and the estimated radiogenic heating.

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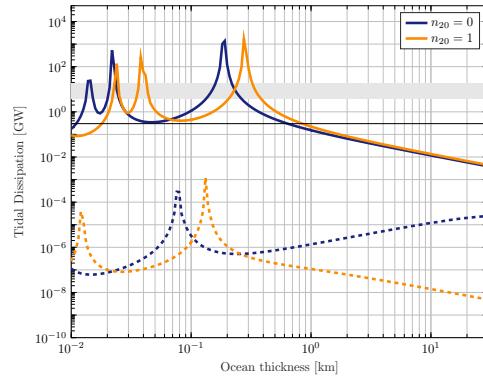


Figure 2: Same as Figure 1 but for different ocean thicknesses and two different ocean topographies. A 1 km thick ice shell and a Rayleigh friction coefficient of  $\alpha = 10^{-7} \text{ s}^{-1}$  has been used