EPSC Abstracts
Vol. 13, EPSC-DPS2019-356-1, 2019
EPSC-DPS Joint Meeting 2019

EPSC
EPSC-DPS Joint Meeting 2019
15–20 September 2019 | Geneva, Switzerland

Resonances and rings around non-axisymmetric bodies

Bruno Sicardy,
LESIA, Observatoire de Paris, Université PSL, CNRS, Sorbonne Université, Univ. Paris Diderot, Sorbonne Paris Cité, 5 place Jules Janssen, 92195 Meudon, France (bruno.sicardy@obspm.fr)

Abstract

We describe various high order resonances between a rotating body and an orbiting particle. The approach encapsulates in a single framework both retrograde and prograde resonances, as well as resonances that occur inside or outside the corotation radius.

We present a simple way to calculate the strengths of those resonances, using and adapting the coefficients that appear in the classical expansion of the disturbing potential caused by a point-like satellite.

Possible effects on rings like those of Chariklo and Haumea are discussed.

1. Introduction

Dense and narrow rings have been discovered in 2013 around the small Centaur object Chariklo [1], and around the dwarf planet Haumea in 2017 [2]. They are the first rings ever observed elsewhere than around giant planets.

Due to their irregular shape, small bodies create strong resonances that should have a significant effect on surrounding rings [3]. Contrarily to the cases of the giant planets, high order resonances must be accounted for around small bodies. Here we present a simple way to calculate the strength of high order resonances using the coefficients of the disturbing potential caused by a satellite, as provided in [4].

2. Resonances taxonomy

Resonances take the form $j\kappa = m(n - \omega)$, where κ and n are the epicyclic frequency and the mean motion of the particle, respectively, Ω is the rotation rate of the body, and j and m are integers (with the convention $j > 0$, while m may be positive or negative). Because in general $\kappa \sim n$, they correspond to

$$\frac{n}{\Omega} \sim \frac{m}{m-j}, \tag{1}$$

and are loosely referred to as a $m{:}m-j$ resonances.

The resonances can be prograde ($n/\Omega > 0$) or retrograde ($n/\Omega < 0$). The order of the resonance is j, the case $j = 1$ corresponding to the classical Lindblad resonances. The case $m < 0$ corresponds to prograde outer resonances (i.e. occurring outside the corotation radius), while the case $0 < j < m$ corresponds to prograde inner resonances. The case $m = j$ corresponds to apsidal resonances. Finally, the case $m < j < 2m$ corresponds to retrograde inner resonances and the case $2m < j$ corresponds to retrograde outer resonances.

As shown in [3], a $m{:}m - j$ resonant periodic orbit crosses itself $|m|(j - 1)$ times (if m and j have been reduced to their relatively prime version). This shows that only Lindblad resonant orbits do not cross themselves, so that the corresponding streamline are well-behaved in terms of hydrodynamical evolution. Two resonances cause streamlines with only one self-crossing point, one with $m = -1$, $j = 2$ (the outer 1/3 resonance), and one with $m = +1$, $j = 2$ (the retrograde corotation resonance). Note that both resonances have the same dynamical structure.

3. Resonances strength

Considering a potential of the generic form

$$U(\mathbf{r}) = \sum_{m=-\infty}^{+\infty} U_m(r)\cos(m\theta), \tag{2}$$

the perturbing potential associated with a $m{:}m-j$ resonance has the form

$$U_{m,j} = e^j \bar{U}_{m,j}(a/R)\cos(\varphi_{m,j}), \tag{3}$$

where $\varphi_{m,j} = m\lambda' - (m - j)\lambda - j\varpi$ is the resonant angle, λ' is the orientation angle of the body and R its characteristic size, and a, e, λ and ϖ are the semi-major axis, orbital eccentricity, mean longitude and pericenter of the particle, respectively.

The factor $\bar{U}_{m,j}(\alpha)$ (with $\alpha = a/R$) measures the strength of the $m{:}m - j$ resonance. It is easily derived from U_m through

$$\bar{U}_{m,j} = 2F_n[U_m], \quad (4)$$

where F_n are the operators that appear in the expansion of the Disturbing Function (see [4]).

Table 1: Resonance strengths.

Order j	Resonant potential
1	$2eF_{27}[U_m]\cos[\lambda' - (m-1)\lambda - \varpi]$
2	$2e^2F_{45}[U_m]\cos[\lambda' - (m-2)\lambda - 2\varpi]$
3	$2e^3F_{82}[U_m]\cos[\lambda' - (m-3)\lambda - 3\varpi]$
4	$2e^4F_{90}[U_m]\cos[\lambda' - (m-4)\lambda - 4\varpi]$

From [4], we get $F_{27} = -1/2[2m + \alpha d/d\alpha]$, and more voluminous expressions of the same kind for the operators F_{45}, F_{82} and F_{90}.

In the case of the potential created by a homogeneous triaxial ellipsoid ([3]), the derivative operator $d/d\alpha$ is replaced by an algebraic multiplication, which greatly simplifies the calculation of the resonance strengths.

4. Phase portraits

Representative phase portraits corresponding to $j = 1$ (Lindblad resonances), $j = 2$ and $j = 4$ are sketched in Fig. 1.

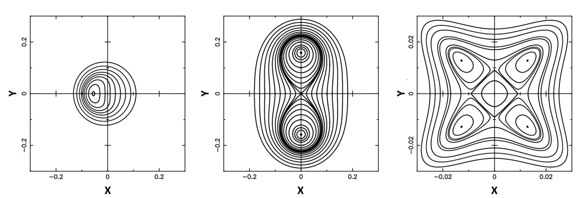

Figure 1: From left to right, phase portraits of first, second and fourth order resonances. Each line is a trajectory of the eccentricity vector $[e\cos(\varphi_{m,j}/j), e\sin(\varphi_{m,j}/j)]$.

The topology of each portrait provides hints about the response of a ring to a resonance. Lindblad resonances (left panel) have a fixed point that is shifted with respect to the origin. This causes a torque between the body and a collisional disk and forces it to migrate. In the case of a 2nd-order resonance (middle panel), the origin is an unstable hyperbolic point, which forces the particles to acquire large orbital eccentricities. Thus 2nd-order resonances are also expected to have a significant effect on the disk. In the case of a 4th-order resonance (right panel), the origin is always a stable equilibrium point. As collisions damp the orbital eccentricities of the particles, these resonances are expected to have little effects on a ring.

5. Summary and Conclusions

Although the expansion of the classical disturbing potential seems restricted to a point-like satellite, it is in fact rather generic. It can be used in particular to described resonances around an irregular rotating body.

We note that high order resonances cause self-crossing streamlines. In that context, simulations using collisional codes will help in the future to understand better how those resonances may sculpt a ring around an irregular body.

Acknowledgements

The work leading to these results has received funding from the European Research Council under the European Community’s H2020 2014-2020 ERC Grant Agreement no 669416 “Lucky Star".

References

[1] Braga-Ribas, F., Sicardy, B., Ortiz, J. L. et al: A ring system detected around the Centaur (10199) Chariklo. Nature Vol. 508, pp. 72-75, 2014.

[2] Ortiz, J. L., Santos-Sanz, P., Sicardy, B., et al: The size, shape, density and ring of the dwarf planet Haumea from a stellar occultation. Nature Vol. 550, pp. 219-223, 2016.

[3] Sicardy, B., Leiva, R., Renner, S. et al.: Ring dynamics around non-axisymmetric bodies with application to Chariklo and Haumea, Nature Astronomy, Vol. 3, pp. 146-154, 2019.

[4] Murray, C. D. and Dermott, S. F.: Solar System Dynamics, Cambridge University Press, 1999.