

# Study of the dynamics of the 1/3 mean-motion resonance between trans-Neptunian objects and Neptune

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## Abstract

In this work we do an analysis of the dynamics of the 1/3 exterior mean-motion resonance (MMR) between trans-Neptunian objects (TNO) and Neptune. Considering the restricted three body problem, we develop a semi-analytical planar averaged Hamiltonian. With this approach we plot the representative planes as a function of the orbital elements and we are able to find the stationary solutions of the problem. To analyze the dynamics around these stationary solutions we use dynamical maps and check the robustness of these resonance. We then consider the 6-body problem including the perturbation of the inner giant planets and compare with the results obtained from the 3-body problem. We complement the study to the inclined case and verify the results by the use of numerical integrations of the exact equations of motion of the problem.

## 1. Introduction

The dynamical classification of a TNO is usually related to their current dynamical stage, without considering their past or future state [1]. These bodies have a great variety of orbital parameters and to explain them is one of the open problems of the solar system. The comprehension of their formation and motion may give us hints about the late evolution of the solar system and how these bodies got at their current states. Among these bodies, we study the thretninos, which are close to a 1/3 MMR with Neptune. Most of these bodies have an orbital inclination larger than  $20^\circ$  and are likely involved to Lidov-Kozai cycles. A primordial instability on the solar system, with a slow outer migration of Neptune is one of the possible explanations that lead to an overpopulation of the TNOs in resonance [2], and their excited eccentricities and inclinations. Due to that, the resonance is an important mechanism for the stability and present dynamical features of the TNOs.

An object is considered locked to a MMR when there is a commensurability between the mean-

motions of the form:

$$(p + q)n_2 - pn_1 \approx 0, \quad (1)$$

where  $n_i$  are the mean-motion of the bodies, the indices 1 for the inner and 2 for the outer body in relation to the position of the central star,  $p$  and  $q$  are integer numbers, and  $q$  is the order of resonance. That means that the ratio of their orbital periods are close to an integer or semi-integer number, which determines a relation between their mean longitudes. Based on that and by D'Alembert rule, we can define the resonant angle:

$$\sigma = (p + q)\lambda_2 - p\lambda_1 - q\varpi. \quad (2)$$

When in resonance, this angle librates around a fixed value. This behaviour is related to the stationary solutions of the problem, which is called the *The Apsidal Corotation Resonance (ACR)*. When this libration occurs around  $\sigma = 0$  or  $\sigma = \pi$ , we say he have a symmetric ACR. When this oscillation occurs around an angle with value different than those, it is called an Asymmetric ACR.

## 2 Methods and Results

We develop a numerically averaged semi-analytic Hamiltonian for the restrict three-body problem, following the calculations done by [3]. If we consider the eccentricity of Neptune  $e' = 0$ , the problem has a constant of motion and the problem is then reduced to a two-degree of freedom system.

We are able to plot the representative planes as a function of the orbital elements and angles and with numerical algorithms we are able to find the symmetric and asymmetric stationary solutions of the problem for the planar case. Since most of the TNOs close to a 1/3 MMR have inclinations  $I > 20^\circ$ , we also study the the inclined case. We use the angle-action variables described by [4] on our model and find the stationary solutions for these cases as well.

We extend the problem to the case where the perturbation of the inner Giant Planets are considered. We realize an expansion over the Legendre Polynomials averaged over the mean-longitudes and longitudes of pericenters of the giant planets to add the correspondent perturbation terms to our Hamiltonian. Then, we are able to describe the differences in the topology in comparison with the three-body problem. We notice that the libration center are displaced from the nominal resonance value.

The, we develop Dynamical Maps and the evaluation of the Dynamic Power Spectra from a grid of initial conditions around the resonance zone. We are able to evaluate how the main frequencies depends on the orbital elements and how chaotic the orbits are in relation to the initial conditions. We are able to identify the dynamical zones around the resonance for the 1/3 MMR and how the frequencies of the problem behave as a function of the orbital elements (Figure 1) and that the perturbation from the inner giant planets increase the chaos around the resonance.

To check the validity of our model we compare the results with the integration of the Newtonian equations of motion using the Radau-15 Integrator ([5]). We use the data of the objects close to the 1/3 MMR obtained from the Small-Body Database Brouwer (<https://ssd.jpl.nasa.gov/sbdb.cgi#top>) and utilize their parameters to check their dynamical states in comparison to our model. Between the TNOs that we studied, just 2003FZ<sub>129</sub> exhibited a stable libration around a fixed value.

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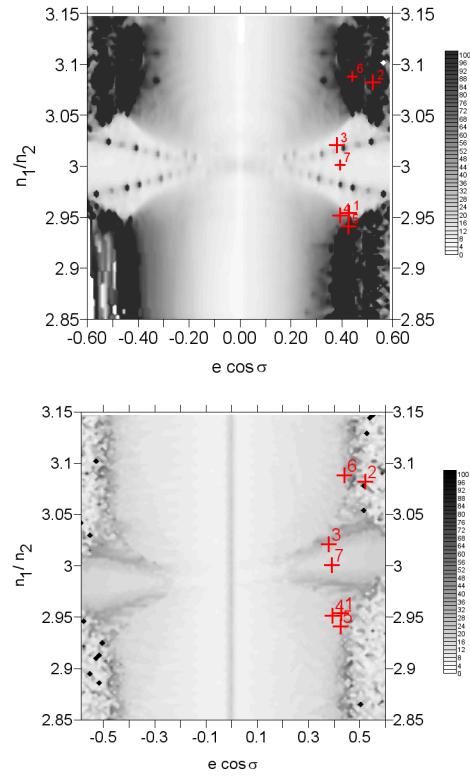


Figure 1: On both graphics the numbers corresponds to the asteroids: 1 - 2004 VU<sub>130</sub>, 2 - 2000 SS<sub>331</sub>, 3 - 2006 HR<sub>122</sub>, 4 - 2005 PQ<sub>21</sub>, 5 - 2004 HF<sub>200</sub>, 6 - 2014 WL<sub>510</sub> and 2003 FZ<sub>129</sub>. Upper Figure: Dynamical map for the restricted 3-bodies problem, the color scale represents the chaoticity measured by the Spectral number. Bottom: Dynamical map for the 6-body restrict planar problem. The darker, the more chaotic is the dynamics.