

Using deep neural networks to compute the mass of forming planets

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Abstract

We present deep neural networks (DNN) in order to predict the critical core mass and the envelope mass of forming planets. We show that our neural networks provide a very good approximation (at the percent level) of the result obtained by solving interior structure equations, but the required computer time is much shorter. The difference with the real solution is much smaller than the difference that is obtained with some analytical formulas that are available in the literature, which only provide the correct order of magnitude at best. We show that these analytical formulas can severely overestimate the mass of planets and therefore predict the formation of planets in the Jupiter-mass regime instead of the Neptune-mass regime. The python tools that we provide allow computing the critical mass and the mass of planetary envelopes in a variety of cases, without the requirement of solving the internal structure equations. These tools can easily replace previous analytical formulas and provide far more accurate results.

1. Introduction

Understanding the formation of planets (from terrestrial ones to gas giant ones) requires the development of theoretical models whose outcome can be compared with observations. One primary outcome of planet formation models is the mass and internal structure (in particular mean density) of planets, as these two quantities can be obtained using radial velocity and transit observation of extrasolar planets. In the core accretion model, the mass growth of planets results from two phenomena. The first is the accretion of solids (planetesimals or pebbles), which depends on the properties of the accreted solids, the planetary mass, and the disc thermodynamical properties (e.g. Fortier et al. 2013). The second one is the accretion of gas (see e.g. Venturini et al. 2016). For sub-critical planets (when

the core mass is smaller than the envelope mass), the computation of the planetary envelope requires solving a set of four differential equations. Although standard, solving these internal structure equations can require a non-negligible amount of computer time, and lead in some cases (e.g. close to the critical core) to some numerical instability. For that reason, some authors have developed fitting formula allowing estimating the envelope mass as a function of some parameters (e.g. opacity, core mass, etc...), as well as some analytical approximation of the critical core mass (e.g. Ikoma et al., 2000, Ida and Lin 2004, Bitsch et al. 2015).

The use of these simplified formulas, although convenient from the numerical point of view, is questionable when computing the formation of low mass planets, up to the Neptune mass range. As an example, in the context of planet formation by pebble accretion, Brügger et al. (2018) compared the resulting mass function obtained using the equation proposed by Bitsch et al. (2015) on one side, and computed solving internal structure equations on the other side. They showed that the resulting planetary mass was much smaller in the latter case.

Here, we focus on sub-critical planets, and we use Deep Neural Networks (DNNs) to compute the envelope mass of forming planets as a function of the relevant parameters. The advantage of such an approach is two fold: first the resulting envelope mass is close to the 'real' one (meaning computed by solving the internal structure differential equations), and second the computer time required to compute these masses is orders of magnitude smaller.

2. DNN architecture

The DNN we consider has 5 hidden layers, all of them having 128 units. These numbers were found by a series of trial and error tests, and we chose one architecture that gives good enough results. We emphasise the

fact that the architecture we present is just one possible, and there are very probably other architecture that would provide even better results. The DNN is fully connected, meaning that each unit of a layer is connected to each layer of the previous and next layer. For each unit, we chose to use the ReLU function for the non-linearity, given by $ReLU(x) = \max(x, 0)$. The cost function we minimise is the mean square error between the predicted and the actual envelope mass.

3. Data

To generate the data, we have in a first step selected ~ 10000 points in a four dimension space, by drawing at random a semi-major axis a (uniform in log between 0.1 and 30 AU), a pressure P (uniform in log between 10^{-2} and 1500 dyn/cm^2), a temperature T (uniform between $30K$ and $1500K$), and a luminosity L (uniform in log between 10^{22} erg/s and 10^{29} erg/s) We emphasise that L is the *total* luminosity, meaning that it has a contribution from both solid accretion and envelope contraction. For each of these (a, L, P, T) , we computed the planetary mass for different planetary cores, solving differential equations presented in Alibert and Venturini (2019).

4. Results

In order to compare the result of the DNN and the one coming from the resolution of internal structure equations, we present in the figure below planetary growth tracks for a planet forming at 5 AU. The solid accretion rate is fixed to $\dot{M}_{\text{core}} = 10^{-6} M_{\oplus} / \text{yr}$. On the figure, we show the evolution of the 'real' envelope mass (obtained by solving internal structure equations), the one predicted by the DNN, and the ones that would be obtained with the formulas of Ikoma et al. (2000) and Bitsch et al. (2015). For these two cases, we consider in each case two possible values of the envelope opacity, namely 1 and $10 \text{ cm}^2/\text{g}$. As can be seen, it is clear that the results of the DNN is much closer to the results obtained by solving the internal structure equations, than by implementing the analytical formulas. It is also notable that the envelope mass is extremely over-estimated by using the equations of Bitsch et al. (2015 - green region).

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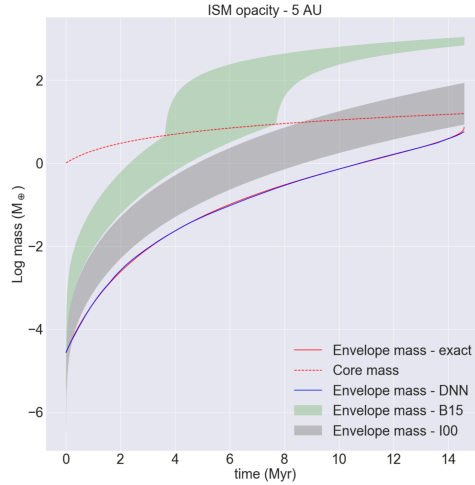


Figure 1: Exemple comparison between results of the DNN (blue line) and analytical formulas by Ikoma et al. (2000 - gray region) and Bitsch et al. (2015 - green region). The grey and green regions corresponds to the values of the envelope mass for different possible opacities. The red dashed line shows the core mass as a function of time which is the same in all the cases, and the red solid line shows the planetary mass obtained by solving internal structure equations (Venturini et al. 2016, Alibert and Venturini, 2019).

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