

Effects of General Relativity on the secular evolution of two planets orbiting close to the star

Francesco Marzari (1), Makiko Nagasawa (2)

(1)Department of Physics and Astronomy, University of Padova, via Marzolo 8, 35131, Padova, Italy (2)Kurume University, School of Medicine, Department of Physics

Abstract

Among the different final architectures of planets after either a chaotic phase of evolution (Planet–Planet scattering) or gas–driven migration, there are cases in which two planets end up orbiting very close to the star. In these configurations, the induced precession of the periapsis of the inner body due to General Relativity may significantly affect the classical secular evolution of the pair by altering both the main secular frequencies and the proper and forced eccentricity values.

1. Introduction

Planet–planet scattering, a mechanism initially invoked to explain the high orbital eccentricities observed among extrasolar planets, has also revealed the ability to drive planets very close to the star, once coupled to tides. Starting from a system of three planets, the two survivors at the end of the chaotic phase may end up on close orbits whose evolution is determined not only by the mutual gravitational interaction but also by the effects of General Relativity (hereinafter GR). In this scenario, the classical secular theory usually adopted to predict their orbital evolution may not be accurate because of the periapsis precession induced by GR. A similar outcome may also be the result of gas driven migration where a pair of planets are taken very close to the star before the dissipation of the gaseous disk.

2.

In the framework of the second order secular theory of Laplace–Lagrange, the equation for the non–singular variables $h_i = e_i \sin(i)$, after the inclusion of the GR term, becomes:

$$\dot{h}_i = k_1 \left(A_{i1} + \frac{3G^3(M_S + m_p)^{\frac{3}{2}}}{a_p^{\frac{5}{2}} c^2 (1 - e_p^2)} \right) + k_2 A_{i2}, \quad (1)$$

where $k_i = e_i \sin(\varpi_i)$, while the matrices $A_{i,j}$ are given in [2] and are only functions of the semimajor axes and masses of the two planets. The additional term due to GR causes a change in the secular frequencies and a reduction of the half-width of the secular oscillation. This theory works only for small eccentricity of the planets. If the eccentricity of the inner planet is high, the situation becomes more complex since the second order secular theory does not properly work at high eccentricities and higher order expansions of the perturbative potential are required as suggested in [1]. In these cases we resort to direct numerical integration of the planet motion.

3. Numerical modeling

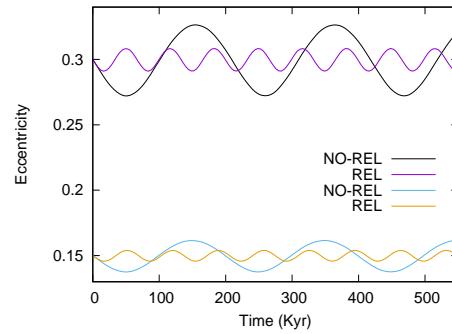


Figure 1: Secular evolution of the eccentricity of the inner planet of a pair whose orbits have been integrated with and without the GR term. The semimajor axis of the inner planet is $a_1 = 0.1$ au while its eccentricity is set to $e_1 = 0.15$ and $e_1 = 0.3$ in the two different cases. The outer planet has $a_2 = 1.5$ au and its eccentricity is $e_2 = 0.1$. The mutual inclination is initially set to 3° .

We have used a numerical integration code based on the Hermite algorithm that includes the effects of gen-

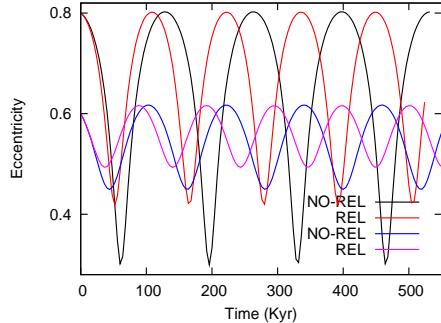


Figure 2: Same as in Figure 1 but the semimajor axis of the inner planet has been shifted outward to $a_1 = 0.2$ au to show that GR is relevant even further out. The initial eccentricities of the planets are $e_1 = 0.6$ and $e_1 = 0.8$ while that of the outer planet is fixed to $e_2 = 0.1$.

eral relativity, computed with the PN approximation, and dynamical tides [3].

In Figure 1 we illustrate the secular behaviour of the inner planet with and without the GR term. The eccentricity is low on average and the behaviour is well described by the Laplace–Lagrange theory and a shrinking of the proper oscillation is observed, as expected. When the eccentricity of the inner planet is raised, as shown in Figure 2, a significant increase of the forced eccentricity is observed, which is not predicted by the classical theory, and, on average, the eccentricity is higher than in the pure N–body case. If the tidal interaction with the star is included in the numerical model, this higher eccentricity leads to a faster inward migration, as shown in Figure 3.

4. Summary and Conclusions

Preliminary calculations have shown that GR significantly affects the secular evolution of two planets orbiting close to the star. While for low eccentricities the behaviour can be interpreted with the inclusion of a diagonal term in the secular theory of Laplace–Lagrange, for high eccentricities the situation is more complex. We will present additional simulations where we will try to formulate a theory able to predict the observed increase of the forced eccentricity of the inner planet. This is an important effect since it leads to a reduction of the tidal migration timescale. In addition, the interpretation of TTV (Transit Time

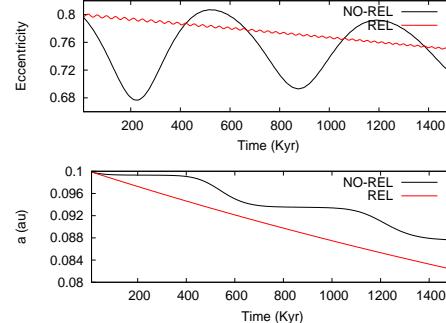


Figure 3: As in Figure 2 but the semimajor axis of the inner planet is set initially to 0.1 au and tidal forces are included.

Variations) data will be more accurate.

References

- [1] Libert, A.-S. and Henrard, J., Analytical Approach to the Secular Behaviour of Exoplanetary Systems, *CMDA* 93, 187, 2005
- [2] Murray, C. D. and Dermott, S. F., *Solar system dynamics*, Cambridge University Press, Cambridge 1999
- [3] Nagasawa, M. and Ida, S., Orbital Distributions of Close-in Planets and Distant Planets Formed by Scattering and Dynamical Tides, *ApJ* 742, 72, 2011