

# The 1D kinetic approach for the self-consistent description of a thin current sheet

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#### Abstract

The self-consistent kinetic description of a thin current sheet, is a fundamental task in the space plasma physics. In presented approach the description of a thin current sheet is based on the analysis of singleparticle motion. In frame of specific conditions regarding the scales of magnetic field and particle motion it was possible to find the asymptotical expression for the conservation of magnetic momentum  $(\mu)$ along the particle trajectory, in frame of angular variables (the particle pitch-angle  $\theta$ , the phase of rotation  $\varphi$ ). The  $\mu$ -conservation was used for description in self-consistent manner the distributions of plasma and magnetic field inside the thin current sheet. The distributions have been confirmed by the simplest modeling of independent moving particles. The coincidence of analytical and numeric approaches demonstrates the high efficiency and potential usefulness of the proposed approach.

### 1. Introduction

In the last decades, the CSs in the Earth magnetosphere are intensively investigated by means of space missions (e.g. Cluster, Geotail, THEMIS, etc.). These observations show that the magnetospheric CS may have a rather wide range of scale from a few radii of Earth up to a few hundred kilometers before substorm onset [1]. A special interest, in that respect, consists in investigations of a thin current sheet (TCS), which has a scale an order of thermal Larmor radius of proton  $\rho_T = V_T / \omega_L$ , where  $\omega_L$  is the Larmor frequency of proton gyration and  $V_T = \sqrt{kT/m_p}$  is proton's thermal speed. Such TCSs are the natural object observed in Terrestial magnetosphere [2], in the solar wind [3], etc., and they are expected to play an important role in structuring the magnetospheres of exoplanets. Therefore, one of the main problem of space plasma consists in the self-consistent description of a TCS fine structure, that can be used for the estimation of the value of conserved magnetic energy. Usually, the latter is characterized by TCS self-thickness, which could be calculated by means of analysis of spacecraft DATA. For instance, the statistical studies of TCSs based on the in-situ spacecraft observations, show that TCSs selfthickness may vary in the range, from one up to 20 thermal radius [4]. Such wide range of scales could be explained by different values of energy of plasma flow incoming the TCS, which can also effect on the topology of TCS, such as bifurcation or asymmetry of an electric current distribution.

# 2 The self-consistent TCS description versus numerical simulation

The self-consistent description of TCS requires the specific relation between the particles impulse (or inducted current **J**) and magnetic field geometry (given in the problem) as  $\nabla \times \mathbf{B} = \mu_0 \mathbf{J}$ . In that respect, the balance equation could be rewritten as

$$\frac{d^2F}{d\xi^2} = \tilde{J}(F,\theta_0),\tag{1}$$

where  $\theta_0$  initial pitch angle,  $\tilde{J}(F, \theta_0)$  contains the  $q \int_{-\infty}^{z} B_x(z') dz'$  conservation of magnetic moment,  $F = \frac{0}{mv_0 \sin \theta_0} d\xi = \frac{dz}{I_l}$  and  $I_l = \sqrt{\frac{m_p}{\mu_0 n_0 q^2}}$  the proton inertial length. The solution of eq. (1) together with boundary conditions  $(F(0) = 0, \frac{dF}{d\xi} \mid_{\xi=0} = 0)$  could be easily written in quadrature form

$$\int_{0}^{F} \frac{d\tilde{F}}{\sqrt{2\int_{0}^{\tilde{F}} \tilde{J}(F',\theta_0)dF'}} = \xi.$$
 (2)



Figure 1: Figure a) represents the distribution of electric current and figure b) represents the number density of the TCS for  $\theta_0 = 0.2$ . The red line is the result of numerical simulation, the blue line is the result of analytical solution by equation (2)

By solving numerically the eq. 2 it is possible to find the self-consistent distributions of the plasma parameters (J,N) and the magnetic field  $\mathbf{B} = [B_x, 0, B_z]$ as a function of running coordinate z. To verify the results of analytical treatment, we performed the numerical simulation of 60 protons approaching the TCS from both sides, where magnetic field geometry is calculated with help of eq. 2. Based on all calculated 120 trajectories the phase space  $\{\theta, \varphi\}$  for the running values of z-coordinate was found, that finally leads that the electric current  $J_y$  and number density N as the functions of z can be calculated as

$$J_y(z) = J_0 \int_{\varphi_{min}(z)}^{\varphi_{max}(z)} \sin^2(\theta(\varphi)) \cos(\varphi) d\varphi, \quad (3)$$

$$N(z) = N_0 \int_{\varphi_{min}(z)}^{\varphi_{max}(z)} \sin(\theta(\varphi)) d\varphi, \qquad (4)$$

where  $\varphi_{min}(z)$  and  $\varphi_{max}(z)$  stay for the boundaries of the restored phase-space for the corresponding z. The dimensional coefficients  $N_0, J_0$  have the following form  $N_0 = \tilde{N}_0/4\pi \sin(\theta_0), J_0 = eV_0N_0 \sin^2(\theta_0)$ , where  $V_0$  and  $\tilde{N}_0$  are module velocity and the number density of incoming flow, respectively.

The comparison of analytical treatment and particles simulations for the value of pitch-angle  $\theta_0 = 0.2$  are presented in Fig. (1). In Fig. (1) only one half-space (e.g.,  $z \ge 0$ ) is presented due to the symmetry of problem.

As one can see in Fig.1, the results of numeric simulation are rather close to the analytical description. The analytical and numerical calculations show that the particle number density and the electric current reach the maximum value not in the center of current sheet (z = 0), but slightly aside of it (at  $z = z_m$ ), i.e., bifurcated current. The obtained splitting (i.e., shifted maximum) of the electric current and particle density profiles is due to the oscillatory character of particle motion in the close vicinity  $(\pm z_m)$  of the TCS.

### 3. Summary and Conclusions

We present the results of analytic treatment of proton dynamics in the vicinity of TCS in frame of magnetic moment conservation. The comparison of selfconsistent current distribution with the corresponding numerical modeling show good qualitative and quantitative agreement. That demonstrates the high efficiency and potential usefulness of the proposed analytical approach for the analysis of TCSs.

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