

The obliquity variation induced by a giant impact: Implication of the origin of the obliquity of Uranus

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Abstract

Uranus and Neptune have similar mean density but different obliquity, which suggest a key of formation history. Since the obliquity difference is caused by a impact event, we investigate the giant impact on the ice giant to reproduce the planetary obliquity. We find that the $1 M_{\oplus}$ impactor can reproduce present obliquity of Uranus. Moreover, the impact event should disappear the pre-impact rotation and give another direction of planetary spin in order to make the large obliquity.

1. Introduction

Uranus and Neptune, which are ice giants in our solar system, have similar mass and radius. On the other hand, those planets have difference in obliquities, satellite systems, and intrinsic luminosities, which implies the different formation history. In this study, we focus on the difference in the obliquities of Uranus and Neptune. Uranus's obliquity is 98° while Neptune is 28° , which suggest that Uranus's obliquity is key to discuss the difference in origin of Neptune. The obliquity variation is caused by two factors; one is the tidal interaction and the other is the giant impact event. The obliquity variation due to the tidal interaction is dominant on terrestrial planets (e.g. Earth, Mars) in our solar system since those planets are influenced by the gravitational perturbation by Sun [1]. However, Uranus are far from the Sun and also Jupiter, the tidal interaction cannot change its obliquity [2]. Thus, Uranus's obliquity is mainly determined by the giant impact event. Previous studies [3, 4] showed that the single giant impact was able to reproduce present rotation period of Uranus, while those studies assumed that the proto-Uranus was non-rotating body. A giant impact changes the internal compositional structure of the planet [5], which also changes the thermal evolution of the planet when the planetary atmosphere is strongly polluted by an icy materials [6]. Therefore, investigating the impact event is important to discuss the origin of ice giants. Here we consider the giant im-

pact on rotating proto-Uranus to calculate the obliquity and investigate the impact condition that reproduce the present Uranus obliquity.

2. Method

We solve the following hydrodynamic equations by use of the Godunov-type smoothed particle hydrodynamical calculation, hereafter GSPH,

$$\frac{d\rho}{dt} = -\rho \nabla \cdot \mathbf{v} \quad (1)$$

$$\frac{d\mathbf{v}}{dt} = -\frac{1}{\rho} \nabla P + \nabla \int dx'^3 \frac{G\rho(x')}{|\mathbf{x} - \mathbf{x}'|} \quad (2)$$

$$\frac{du}{dt} = -\frac{P}{\rho} \nabla \cdot \mathbf{v} \quad (3)$$

$$P = P(\rho, u) \quad (4)$$

where ρ , P , \mathbf{v} and u are density, pressure, velocity, and specific internal energy, respectively. t is the time, \mathbf{x} is the position, and $G(= 6.67408 \times 10^{-8} \text{ cm}^3 \text{ g}^{-1} \text{ s}^{-2})$ is the gravitational constant. The detail of our numerical method is described in [7].

Properties of the target and the impactor are shown as Table 1. We derive the shape of the rotating body as follows. We calculate the rotating target whose rotation period is 12 hours as a rigid body. The shape of the rotating body become stable and we adopt it as the initial condition of the rotating target.

We set the impact velocity (hereafter v_{imp}) is equal to the escape velocity (hereafter v_{esc});

$$v_{\text{imp}} = v_{\text{esc}} = \sqrt{\frac{G(M_{\text{target}} + M_{\text{impactor}})}{(R_{\text{target}} + R_{\text{impactor}})}} \quad (5)$$

where M_{target} is the target's mass, M_{impactor} is the impactor's mass. R_{target} is the target's radius, and M_{impactor} is the impactor's radius, respectively. The definition of the impact angle is shown in Fig. 1 (a) and (b). The range of impact angle φ is $\varphi = [-50^{\circ}, 50^{\circ}]$. The range of impact angle θ is $\theta = [0^{\circ}, 50^{\circ}]$ for $\varphi = [-50^{\circ}, 0]$ and $\theta = [0^{\circ}, 40^{\circ}]$ for $\varphi = [0^{\circ}, 50]$.

Table 1: Properties of the target and the impactor.

Properties	Target	Impactor
Mass [M_{\oplus}]	13.0	1.0
H_2 [%]	20	0
H_2O [%]	80	100
Period [hour]	12.0	0

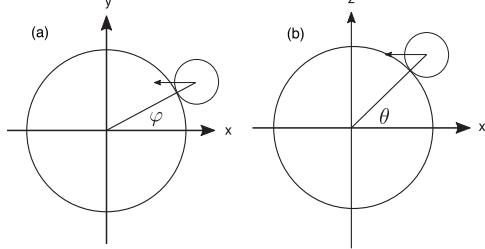


Figure 1: The definitions of the impact angle. (a) shows the impact angle φ on xy plane, while (b) shows the impact angle θ on xz plane.

3. Results

The planetary obliquity is determined by the target's angular momentum after the impact. We define the obliquity Θ as the angle of the angular momentum vectors between the pre-impact target's $\mathbf{L}_{t,0}$ and the target after the impact \mathbf{L}_t . Thus the obliquity is shown as

$$\Theta = \cos^{-1} \left(\frac{\mathbf{L}_{t,0} \cdot \mathbf{L}_t}{|\mathbf{L}_{t,0}| |\mathbf{L}_t|} \right). \quad (6)$$

Here we define the target as the particles that satisfy the condition:

$$\frac{1}{2} m_i v_i^2 - \sum_{i \neq j} \frac{G m_i m_j}{|\mathbf{x}_i - \mathbf{x}_j|} < 0, \quad (7)$$

where m is the particle's mass and v is the particle's velocity, respectively. The subscripts i, j are the particle's number. Eq. 7 represents the particle which is bounded by the gravity.

Figure 2 shows the result of the target's obliquity after the giant impact. We can find that the large obliquity appears only if when the impactor collides with anti-rotation direction.

4. Summary and Conclusions

We investigate the obliquity variation due to a giant impact. We execute the SPH simulation to demon-

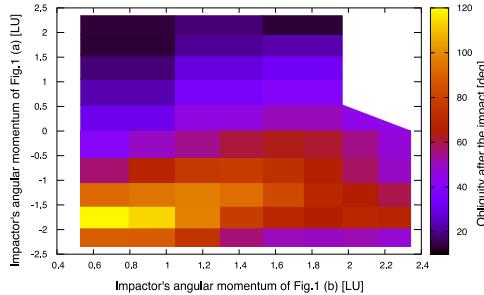


Figure 2: The obliquity after the impact. The impactor's angular momentum is normalized by present angular momentum of Uranus L_U .

strate the large obliquity by a giant impact on the rotating proto-Uranus. We find that $1 M_{\oplus}$ impactor can reproduce the present Uranus's obliquity. In order to make the large obliquity, the impact event should disappear the pre-impact rotation and give another direction of planetary spin.

Acknowledgements

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