

## Using Mixture Density Networks to infer the interior structure of Super-Earths and sub-Neptunes

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### Abstract

We explore the application of Mixture Density Neural Networks (MDN) to the interior characterization of exoplanets. Using a 1D interior structure code, we construct a large training set of synthetic Super-Earths and sub-Neptunian exoplanets up to 25 Earth masses. A model planet consists of an iron-rich core, a silicate mantle, a water ice layer, and an H/He envelope, all constrained by prescribed mass fractions. Using a mixture density network trained on a large data set of such modelled planets, we show to what extent the distribution of possible model solutions can be predicted from just mass and radius. Furthermore, we show that the inclusion of the fluid Love number  $k_2$ , bearing information on the mass distribution in the interior, can significantly increase the prediction accuracy of the model.

### 1. Introduction

The interior characterization of observed exoplanets is one of the main goals in current exoplanetary science. With the large number of newly discovered exoplanets expected in the next ten years by TESS [1] and the upcoming PLATO [2] and JWST [3] missions, a rapid characterization scheme of the interior structure of these planets will become increasingly necessary to further our understanding of planetary populations.

A common approach to the interior characterization of exoplanets is the use of numerical models to compute interior structures which comply with the measured mass and radius of the planet [6]. With only these two observables, possible solutions tend to be highly degenerate, with multiple, qualitatively different interior compositions that can match the observations equally well. As this is an inverse problem, it usually necessitates the calculation of a large number of interior models to obtain an overview over possi-

ble interior structures, which can be computationally expensive, in particular if more observables than just mass and radius are used due to the higher dimensionality of the problem. An additional potential observable, which could help break the degeneracy, is the fluid Love number  $k_2$ , bearing information on the mass concentration in the interior of the planet [4].

In this study, we apply a Deep Learning method to the interior structure calculation of exoplanets based on observed mass and radius. Instead of calculating the interior structure for every planet individually, we initially create a large data set of synthetic planets and train a mixture density network (MDN, [5]) to infer the distribution of the thickness of all planetary interior layers. Mixture density networks are a special case of neural networks, which predict the parameters of a Gaussian mixture distribution instead of single output values.

### 2. Model Setup

We use our interior structure model TATOOINE (Tool for Atmospheres, Outgassing and Optimal Interiors of Exoplanets) to construct a large set of synthetic planets to train a Mixture Density Network. Each modeled planet consists of an iron-rich core, a silicate mantle, an high-pressure ice layer, and a H/He gaseous envelope with solar-like composition. The mass of each layer is constrained by a predefined mass fraction. The mass fractions of all layers add up to one. The core and mantle materials are assumed to have an Earth-like mineralogical composition.

We follow an approach by Bishop (1994) [5] to construct a Deep Learning Mixture Density Network (MDN) to approximate the range of interior solutions for a given mass and radius of the target planet. At its core, the MDN consists of a conventional feed-forward neural network. In contrast to a conventional neural network, however, which maps the input val-

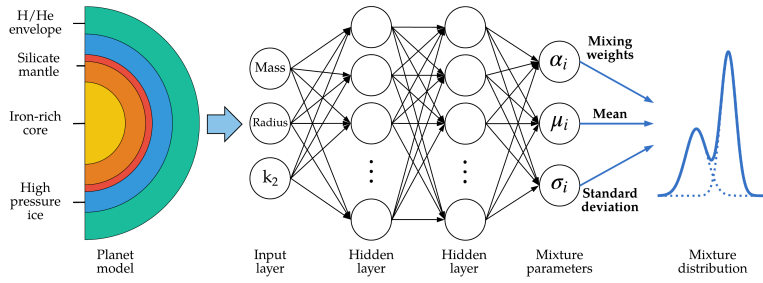


Figure 1: Schematic of the mixture density network architecture used in this work. A large training set of synthetic planets is used to train an MDN with 2 hidden layers. The MDN predicts the parameters of a Gaussian Mixture model corresponding to the probability distribution of interior solutions fitting the input parameters.

ues to discrete output values, the output layer in an MDN uses a Gaussian Mixture model as output, predicting the parameters of the probability distribution of the input data. In our case, the MDN is trained to predict the relative thickness of each interior layer. We use an MDN with 2 hidden layers of 512 neurons each. We explore two scenarios concerning the input variables: having only mass and radius as inputs, and having mass, radius, and the fluid Love number  $k_2$  as inputs.

### 3. Results

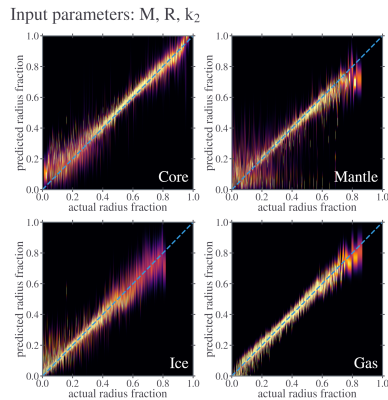


Figure 2: Predicted layer thickness distribution plotted against the actual value from validation data for each of the four interior layers when using mass, radius, and  $k_2$  as network inputs. A perfectly accurate MDN would plot on the dashed blue line.

We find that the MDN trained on mass and radius is able to infer the distribution of the core size and en-

velope thickness fairly close to the distributions obtained by conventional Monte-Carlo sampling. However, it tends to underestimate the thickness of the high-pressure ice layer, and fails to predict the thickness of the silicate mantle accurately.

Using mass, radius, and  $k_2$ , the prediction accuracy of every layer increases significantly (see Fig.2). For the Earth, the MDN can predict the correct interior structure to within a few percent of the real value.

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