

# The Definition and Implications of the Rotational Roche Lobe

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## Abstract

The rotational Roche Lobe (the “lobe”) for an asteroid system was first defined in [1] and since has been computed for a number of asteroid systems. Its geophysical relevance has only been shown recently, however, with the asteroid Bennu having a distinct change in its surface slope properties at the current boundary of its lobe [2]. As the lobe is not a well known concept, this contribution will review the definition of the lobe, provide details on its dynamical relevance and show example computations of its relevance in action.

## 1. Introduction

The rotational Roche Lobe (lobe) for an asteroid was first introduced in an analysis of the asteroid Castalia [1]. It is strictly defined as the zero-velocity surface associated with the lowest energy equilibrium point about a rotating body. It was used to define the maximum possible surface ejection speed for a particle that would guarantee that it would remain trapped to the body. It has been computed for many asteroid systems since this initial definition, and over time the more specific name “rotational Roche Lobe” was used to more easily relate it to other common geophysical parameters, more usually defined for planetary satellites.

The recently published results from the OSIRIS-REx mission [3] made the first documented connection between the rotational Roche Lobe and the surface properties of an asteroid, as Bennu was seen to have a distinct transition in its surface slope distribution at the point where the lobe intersected the asteroid surface [2]. The current abstract is motivated by this observation, with the purpose being to clearly define and introduce this concept to the community, show how it has been applied in the past, and present dynamical computations that illustrate its geophysical role at an asteroid.

## 2 Definition

In a frame that uniformly rotates with the asteroid, motion close to the surface can be analyzed by studying the geopotential  $V(x, y, z) = \frac{1}{2}\omega^2(x^2 + y^2) +$

$\mathcal{U}(x, y, z)$ , where  $x$  and  $y$  are in the asteroid equator,  $z$  is measured along the rotation pole,  $\omega$  is the asteroid spin rate and  $\mathcal{U}$  is its gravitational acceleration potential (the negative of its potential energy). The total energy of the particle in the rotating frame is  $J = \frac{1}{2}(\dot{x}^2 + \dot{y}^2 + \dot{z}^2) - V$ , and is a conserved quantity. If the body does not spin too rapidly, there will be at least 4 and up to many so-called “stationary points” about the body, defined as locations where the gravitational and centripetal accelerations are balanced in the rotating frame. The simple condition for these to exist is that  $V$  take on an extremum, or  $V_x = V_y = V_z = 0$  at a given point. If  $C_{min}$  is defined as the minimum value of the geopotential across all of the stationary points, then the rotational Roche Lobe is defined as the surface that satisfies  $V(x, y, z) = -C_{min}$ , which in general is a 2-dimensional surface with a singular point at its defining stationary point. Figure 1 shows the rotational Roche Lobe about the asteroid Eros.

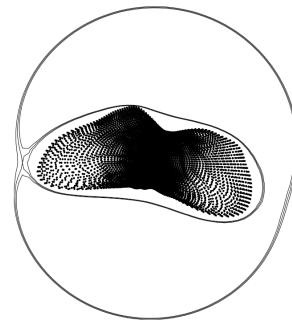


Figure 1: Asteroid Eros with its enveloping rotational Roche Lobe [4]. As the lobe completely envelopes Eros, that asteroid retains more of its loose regolith.

## 3 Dynamical Properties

Every particle in orbit about an asteroid or ejected from its surface has a characteristic energy defined by the function  $J = C$ , if  $C < 0$  then associated with this energy is a zero-velocity surface, defined as the surface such that  $C = -V(x, y, z)$ . Thus, a particle that has an energy  $C$  and with a location on this zero-velocity

surface must have a “zero velocity” and be instantaneously stationary in the rotating frame. This surface acts as an energetic barrier across which a particle cannot cross, thus the surface will delineate regions of allowable motion at a given energy from regions that are excluded. The rotational Roche Lobe is such a surface, however since it is defined at an equilibrium point if the energy is slightly higher than its value the surface goes through a topological change and “opens up,” meaning that a region where particle trajectories can pass is now defined. As the energy increases further these regions grow in size. Conversely, for any energy less than  $C_{min}$  a barrier exists that separates the asteroid surface from the rest of space. This property can be used to define the “necessary escape speed” at any point on the asteroid surface, the speed for a particle to have sufficient energy to open up the lobe in the vicinity of its defining stationary point, computed as  $v = \sqrt{2(C_{min} + V)}$ , where  $v$  is the total speed of the particle relative to the rotating frame. If a particle has a speed greater than this there is no guarantee that it will leave the surface region, although it has sufficient energy to do so. The higher the particle speed is above this limiting value, the more likely escape from the surface region is. If the speed is less than this value, then the particle is definitely trapped to the surface.

## 4 Sensitivity to Spin Rate

A topic of particular interest is how the lobe changes as a function of asteroid spin. For Bennu this is a specifically interesting point, given its documented acceleration in spin rate, which will lead to a doubling in 1.5 Myr [5]. In general, the lobe can have three different topologies about an asteroid. The most common in well studied asteroids is for the entire body to be surrounded by the lobe, meaning that at every point of the asteroid some energy is required in order for a surface particle to escape. At the other extreme, an asteroid rotating beyond its surface disruption rate will not have an enclosing lobe at all, with all surface particles having sufficient energy to escape from the surface [6]. For asteroids rotating rapidly, but not beyond the surface disruption rate, the rotational Roche Lobe can also trap part of the body to the surface, and have other regions of the surface exposed and with sufficient energy to escape. Fast spinning, top-shaped asteroids such as Bennu and 1999 KW4 have this lobe morphology, leading to preferential trapping of material inside the lobe.

## 5 Dynamical Simulations

Our presentation will also show dynamical simulations of particles ejected from an asteroid surface at different energies, both within and outside of the lobe. The simulations will leverage our existing granular mechanics code [7] to illustrate particle ejection and trapping as a function of an asteroid spin rate and lobe. Example computations are shown in Fig. 2.

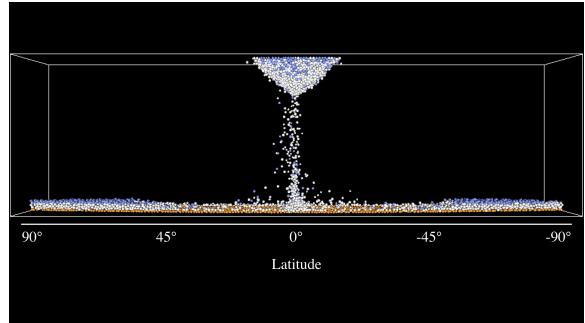


Figure 2: Example computation of landsliding particle motion. The lobe exists as an isolated region near latitude of  $0^\circ$ , where some material remains trapped.

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## References

- [1] D.J. Scheeres, et al. : “Orbits Close to Asteroid 4769 Castalia,” *Icarus* 121:67–87, 1996.
- [2] D.J. Scheeres, et al. : “The dynamic geophysical environment of (101955) Bennu based on OSIRIS-REX measurements,” *Nature Astronomy* 3, 352–361, 2019.
- [3] D.S. Lauretta, et al. : “The Unexpected Surface of Asteroid (101955) Bennu,” *Nature* 568, 55–60, 2019.
- [4] J.K. Miller, et al. : “Determination of shape, gravity and rotational state of Asteroid 433 Eros,” *Icarus* 155, 3–17, 2002.
- [5] C. Hergenrother, et al.: “The Operational Environment and Rotational Acceleration of Asteroid (101955) Bennu from OSIRIS-REx Observations,” *Nature Communications* 10, 1291, 2019.
- [6] D.J. Scheeres. “Landslides and Mass Shedding on Spinning Spheroidal Asteroids,” *Icarus* 247, 1–17, 2015.
- [7] P. Sánchez and D.J. Scheeres. “Cohesive Regolith on Fast Rotating Asteroids,” under review, 2019.