EPSC Abstracts
Vol. 13, EPSC-DPS2019-798-1, 2019
EPSC-DPS Joint Meeting 2019

EPSC
EPSC-DPS Joint Meeting 2019
15–20 September 2019 | Geneva, Switzerland

Planet-planet tides in Kaula's theory

M. Walterová and M. Běhounková
Charles University, Faculty of Mathematics and Physics, Department of Geophysics, Czech Republic
(kanova@karel.troja.mff.cuni.cz)

Abstract

Even a brief exploration of any exoplanetary database gives us the impression that multiplanetary systems are far from being unique. Out of 2936 confirmed planet-hosting stars, 659 are known to be orbited by more than one planet [1] and the largest exoplanetary system known so far hosts as much as eight planetary bodies. As the indirect discovery techniques are most sensitive to planets orbiting close to their host star, a number of the detected exoplanetary systems are relatively tightly packed and the mutual encounters of their members give rise to non-negligible gravitational interactions. Aside from third-body perturbations to the Keplerian motion of each planet, their spin rates and heat states may be affected by mutual tides.

First estimates of the magnitude of planet-planet tides in the TRAPPIST-1 system, including discussion of their effects on planetary habitability, were given in [2] and [3], and the first mathematical model of mutual tidal interaction between planets was published earlier this year by Hay and Matsuyama [4].

Here, we present an alternative approach, based on an extended version of the Darwin-Kaula expansion of tidal potential, and verify the results of [4]. In addition to calculating the contribution of planet-planet (or moon-moon) tides to the energy dissipation, we also plan to explore the possible effect of mutual tidal interactions on the long- and short-term evolution of planetary rotation.

1. Model and methods

Presented derivation of the planet-planet tidal potential is entirely inspired by the serie of steps descibed in [5]. Our goal is to express the external potential in the form of harmonic series: spherical harmonics in the space domain and Fourier series in the time domain. This is accomplished by rewriting the general expression for the external potential due to a perturber with coordinates (r, ϑ, φ) relative to the studied planet [5],

$$\mathcal{U} = \sum_{l=0}^{\infty} \frac{1}{r^{l+1}} \sum_{m=0}^{l} \mathcal{P}_{lm}(\cos\vartheta) \left(\mathcal{A}_{lm}\cos m\varphi + \mathcal{B}_{lm}\sin m\varphi\right), \tag{1}$$

as a function of the Keplerian elements, the orientation of planet's rotation axis, and time. The instantaneous distance between the two bodies can be written as

$$r = \sqrt{r_A^2 + r_B^2 - 2\, r_A\, r_B\, \cos\mathcal{E}}\,, \tag{2}$$

where $\mathcal{E}$ symbolizes mutual angular distance as observed from the host star,

$$\begin{aligned}\cos\mathcal{E} &= \cos(v_B + \omega_B)\cos(v_A - \Omega_B) \\ &\quad + \cos i_B \sin(v_B + \omega_B)\sin(v_A - \Omega_B)\,,\end{aligned} \tag{3}$$

variables i_B, ω_B and Ω_B are angular Keplerian elements of the perturber's orbit with respect to the studied planet's orbit, v_i (i = A, B) stand for the true anomalies of both planets and r_i is the instantaneous distance of planet i from the primary. The instantaneous coordinates of the 'sub-perturber' point on the studied planet's surface can be found by performing a transformation of coordinates from the reference system defined with respect to the perturber's orbit to a reference system of the studied planet.

Once the expressions for the instantaneous planetocentric coordinates are known, we expand (1) to the desired form by repeated use of the multinomial theorem and the trigonometric identities derived in [5]. Finally, we perform the elliptic expansion of

$$\left(\frac{r_i}{a_i}\right)^k e^{ijv_i} = \sum_{q=-\infty}^{\infty} X_q^{k,j}(e_i) e^{iqM_i}\,. \tag{4}$$

where $X_k^{b,c}$ denote Hansen's coefficients, a_i and e_i are the semi-major axis and the eccentricity of the respective planet and M_i denotes mean anomalies. The general form of the resulting potential can then be written as

$$\mathcal{U}_{lm} = \frac{1}{a_>^{l+1}} \sum_{\text{indices}} \left(\frac{a_<}{a_>}\right)^{K_1} \mathcal{F}_{I_2}(i_B, \beta) \times$$
$$\times \mathcal{G}_{I_3}(e_A)\, \mathcal{G}_{I_4}(e_B)\, \mathcal{S}_{I_4}(\omega_B, \Omega_B, \alpha, \gamma, M_A, M_B, \theta) \,, \tag{5}$$

with $\mathcal{F}_{I_2}$ being an analogy of Kaula's inclination functions and $\mathcal{G}_{I_{3,4}}$ standing for corresponding Hansen coefficients (the eccentricity functions) [5]. The harmonic term, S_{I_4}, is a function of the orientation of perturber's orbit (ω_B, Ω_B), the orientation of studied planet's spin axis (α, γ; without the obliquity β), instantaneous position of both planets in the orbit, and the sidereal time θ. The symbol K_1 indicates a combination of indices, while $I_{2,3,4}$ stand for corresponding multiindices.

Tidal heating due to planet-planet loading is then computed by an extended version of the method described by [6], where we assume that the planet's reaction can be characterized by either the Maxwell or the Andrade rheology. The results are also tested against a numerical model of a viscoelastic planet deformed by external force.

2. Preliminary results

We applied the analytical model on the calculation of planet-planet tidal heating in three systems of interest: the tightly packed planetary system of TRAPPIST-1, the large and mid-sized Saturnian moons and the Galilean satellites of Jupiter. In neither of these cases is the planet-planet component of tidal dissipation particularly important, compared to the heat generated by radiogenic sources and tides induced by the primary.

In the case of TRAPPIST-1, the most interesting pair of tidally interacting bodies are planets f and g. For the two limit cases of [4], assuming homogeneous interiors with average viscosity of 10^{21} Pa s (Figure 1) and 10^{14} Pa s governed by the Maxwell rheology, we found that the planet-planet tides account for 8% and 80% of the tidal heating due to the primary, respectively. Among the Saturnian moons, the satellite being affected relatively the most by the moon-moon tides is Tethys. As a result of its extremely low eccentricity, Tethys is only mildly heated by the eccentricity tides raised by Saturn and, as a consequence, the contribution of the tidal dissipation due to Dione is as high as 1% of the Saturnian tides. Finally, the moon-moon tides in the system of Galilean satellites are negligible, compared to the role of Jupiter, and their relative magnitude never exceeds 1%.

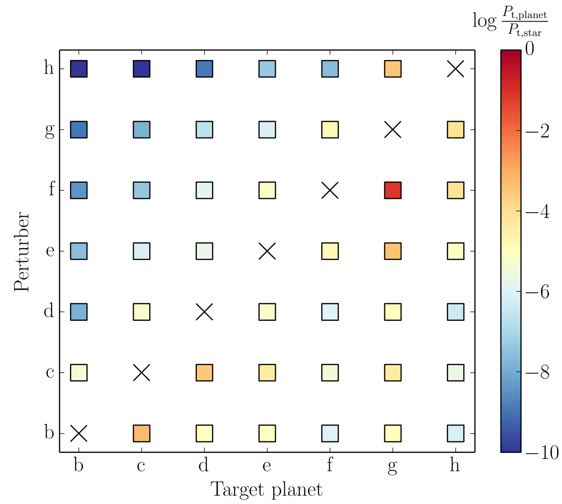

Figure 1: Global heat rate due to planet-planet tides at each planet of the TRAPPIST-1 system, compared to the tidal heating due to primary. Maxwell rheology, $\eta = 10^{21}$ Pa s.

3. Summary and Conclusions

Planet-planet tides may be of slight importance on remote satellites with low eccentricity and obliquity, which are less affected by the loading due to the primary. Although their effect among planetary satellites in the Solar System is negligible, they may present a considerable source of energy in some of the tightly packed exoplanetary systems.

Acknowledgements

This work was supported by the Czech Science Foundation project No. 19-10809S and by the Charles University grant SVV 260447.

References

[1] NASA Exoplanet Archive. (2019, May 4). *https://exoplanetarchive.ipac.caltech.edu*

[2] Lingam, M. and Loeb, A. (2018) *Astrobiology* 18, pp. 967–982.

[3] Wright, J. T. (2018) *Res. Notes AAS* 2, p. 175.

[4] Hay, H. and Matsuyama, I. (2019) *Astrophys. J.* 875.

[5] Kaula, W. M. (1964) *Rev. Geophys.* 2, pp. 661-685.

[6] Segatz, M., et al. (1988) *Icarus* 75, pp. 187-206.