EPSC Abstracts
Vol. 13, EPSC-DPS2019-829-1, 2019
EPSC-DPS Joint Meeting 2019

EPSC
EPSC-DPS Joint Meeting 2019
15–20 September 2019 | Geneva, Switzerland

Airy isostasy on icy moons: Assessment of different approaches

Ondřej Čadek, **Ondřej Souček** and Marie Běhounková
Faculty of Mathematics and Physics, Charles University, Prague, Czech Republic (ondrej.cadek@mff.cuni.cz)

Abstract

We have examined the accuracy of different approaches to modeling Airy isostasy by comparing their results with those obtained from the numerical solution of the equations for viscous flow in the shell of an icy moon with a subsurface ocean. We find that the traditional approach to Airy isostasy, based on simple application of Archimedes' principle, provides a satisfactory estimate of crustal thickness variations at low harmonic degrees. The minimum stress approach, originally proposed by Dahlen (1981) and recently applied to icy moons by Beuthe et al. (2016), leads to results that closely (to within numerical accuracy) match those obtained from the steady-state solution of flow equations. The least satisfactory results are obtained using the equal pressure approach recently proposed by Hemingway and Matsuyama (2017) as an alternative to the traditional approach. The equal pressure approach significantly overestimates the amplitude of the surface topography for all harmonic degrees, suggesting that the deviatoric stress cannot be neglected in the analysis of topographic data.

1. Introduction

In the last few decades, a number of researchers have used Airy isostasy to constrain the internal structure of icy bodies and to determine the variations in the thickness of their outer shells from gravity and topography data collected by space missions. Considerable attention has been paid to icy moons that are thought to have subsurface oceans and which are promising targets for the search for life in the outer Solar System. Recently, two papers have challenged the validity of the traditional (sometimes called "equal mass") approach to isostasy. First, Beuthe al. (2016) proposed a generalization of the concept of isostasy to dynamic systems following the minimum stress approach introduced by Dahlen (1981). A year later, Hemingway and Matsuyama (2017) published a paper in which they presented a new concept of isostasy and argued that the traditional model leads to biased estimates of crustal thickness when applied to small bodies with a relatively thick outer shell. We review these two new concepts, assess their validity and discuss the limits of their applicability to icy moons with a subsurface ocean.

2. Airy Isostasy

The *traditional approach* to Airy isostasy is based on the application of Archimedes' principle to crustal blocks. In spherical geometry, this approach leads to the following relationship:

$$\rho_1 g_1 t_1 R_1^2 = -(\rho_2 - \rho_1) g_2 t_2 R_2^2, \tag{1}$$

where ρ_1, t_1, R_1 and g_1 are the density of the ice crust, the surface topography, the mean radius of the surface and the gravity acceleration at radius R_1, respectively, while the symbols with subscript 2 refer to the same quantities for the ocean. The validity of Eq. (1) has been contested by Hemingway and Matsuyama (2017) who argue that in spherical geometry the standard approach "leads to significant lateral pressure gradients along internal equipotential surfaces and thus corresponds to a state of disequilibrium". They propose an alternative model (hereinafter referred to as the *equal pressure approach*) that is similar to Eq. (1) but does not include the factor of R^2:

$$\rho_1 g_1 t_1 = -(\rho_2 - \rho_1) g_2 t_2. \tag{2}$$

The equal pressure approach assumes that the deviatoric stress in the crust can be neglected. The difference between Eqs. (1) and (2) is usually negligible in the case of the Earth, where R_2^2/R_1^2 is close to 1, but can be significant for bodies like the Moon, Mercury, Mars, Pluto, Ceres or Enceladus where the thickness of the outer shell is not small compared to the mean radius.

Following the work of Dahlen (1981), Beuthe et al. (2016) has recently come up with the concept of dynamic isostasy (hereinafter referred to as the *minimum stress approach*). According to this concept, the crust responds to loading by surface and internal deformation and, after some time, reaches a steady state characterized by the deviatoric stress that is the minimum necessary to support the topography. Our aim is to compare the accuracy of the three approaches described above and determine the one that provides the best prediction of lateral variations in ice shell thickness.

3. Testing the Accuracy

To assess the accuracy of different approaches to isostasy, we compute the steady-state viscous flow in the ice shell driven by variations in hydrostatic pressure along its bottom boundary. Calculations are performed for a shell that has the same size as the ice shell of Enceladus, a small moon of Saturn discussed in recent studies of isostasy. We assume that the ice shell is homogeneous and the viscosity of ice varies with temperature. The relationship between the topographic amplitudes is represented by the load ratio C_ℓ, defined here as follows:

$$C_\ell = -\frac{t_{1,\ell}\,\rho_1\,g_1\,R_1^2}{t_{2,\ell}\,(\rho_2-\rho_1)\,g_2\,R_2^2}, \tag{3}$$

where ℓ denotes the harmonic degree. The load ratio is normalized so that $C_\ell = 1$ for the traditional approach and $C_\ell = (R_1/R_2)^2$ for the equal pressure approach. The results obtained using the different methods are plotted as a function of degree ℓ in Fig. 1.

4. Summary and Conclusions

The load ratio computed from the flow equations (red circles) is close to 1, indicating that the traditional approach (blue squares) is a good approximation of the real ice shell at low harmonic degrees. The primary advantage of the traditional approach is its simplicity. It does not require a priori knowledge of the viscosity profile, nor does it require advanced numerical tools. The equal pressure approach (green diamonds) is comparably simple, but is less accurate leading to results that are significantly different from those obtained using other methods. This approach overestimates the amplitude of the surface topography for all harmonic degrees, suggesting that the assumption of zero deviatoric stress made by Hemingway and Matsuyama (2017) may not be correct. Unlike the traditional model, whose accuracy depends on degree ℓ, the minimum stress model (black triangles) accurately

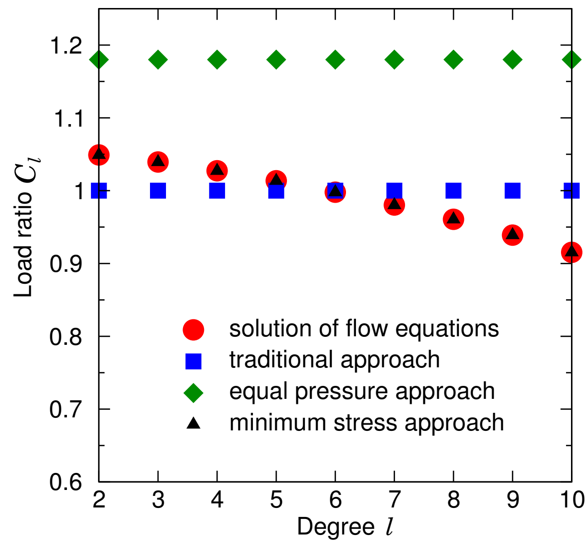

Figure 1: Load ratio C_ℓ as a function of degree ℓ

matches the steady-state solution for all harmonic degrees. This might suggest that the minimum stress approach is equivalent to the solution of the flow equations. However, a careful analysis of the equations shows that these two approaches are equivalent only if the viscosity of the ice shell is constant. For variable viscosity, the minimization of the deviatoric stress should be replaced by the minimization of dissipation.

Acknowledgements

The authors acknowledge the financial support from the Charles University research program UNCE/SCI/023 and the Czech Science Foundation project No. 19-10809S.

References

[1] Beuthe, M., Rivoldini, A., and Trinh, A.: Enceladus's and Dione's floating ice shells supported by minimum stress isostasy, Geophys. Res. Lett., Vol. 43, 10,088-10,096, 2016.

[2] Dahlen, F. A.: Isostasy and the ambient state of stress in the oceanic lithosphere, J. Geophys. Res., Vol. 86, 7801-7807, 1981.

[3] Hemingway, D. J., and Matsuyama, I.: Isostatic equilibrium in spherical coordinates and implications for crustal thickness on the Moon, Mars, Enceladus, and elsewhere, Geophys. Res. Lett., Vol. 44, 7695-7705, 2017.