EPSC Abstracts
Vol. 13, EPSC-DPS2019-887-1, 2019
EPSC-DPS Joint Meeting 2019

EPSC
EPSC-DPS Joint Meeting 2019
15–20 September 2019 | Geneva, Switzerland

Variational Principle for Self-Gravity Wakes and Spiral Density Waves in Planetary Rings

Glen Stewart
Laboratory for Atmospheric and Space Physics, University of Colorado, Boulder, Colorado, USA
(gstewart@lasp.colorado.edu)

Abstract

An important limitation of the Streamline Model for density waves in planetary rings is the complete neglect of local gravitational instabilities which lead to the formation of self-gravity wakes in the A and B rings of Saturn as well as the formation of straw-like textures in the troughs of strong density waves and super-sized self-gravity wakes near satellite-perturbed ring edges, such as the Encke gap edge and the outer A and B ring edges [1], [2]. These local structures may be much more efficient at transporting angular momentum than binary particle collisions and are therefore critical for understanding ring dynamics. A new variational principle for density waves is presented that can also describe local gravitational instabilities in planetary rings

1. Introduction

The goal of this research is to extend the Streamline Model [3], [4] for density waves to include local gravitational instabilities. This has been achieved by modifying a variational principle derived by Alain Brizard for gyrokinetic plasma physics problems [5], [6], [7]. The objective of gyrokinetic theory is to obtain a reduced description of the plasma where the fast gyro-motion of charged particles about the magnetic field has been "averaged out" or, more precisely, transformed away by using Lie-transform based Hamiltonian perturbation theory. The planetary rings version of the variational principle uses the same Lie-transform method to transform away the fast orbital motion of the ring particles in order to obtain a reduced description of the dynamics that is similar, but more general than the traditional Streamline Model.

2. Variational Principle

The dynamics of a particle near an inner Lindblad resonance is expressed in terms of an action-angle Hamiltonian,

$$H = H_0(\Lambda,\mu) + H_1(\lambda,\zeta,\Lambda,\mu) \qquad (1)$$

where the fast angle λ is the mean longitude and the slow angle ζ is the resonant argument for the resonance. The corresponding actions are the radial distance Λ from resonance and the radial action μ associated with orbital eccentricity. After the λ-dependent terms have been transformed away, the resulting action has the form,

$$A = \int dt \int d\zeta\, d\mu \int d\Lambda \left[\tfrac{1}{2}\{S_{dw}, f_0\}\left(\frac{\partial S_{dw}}{\partial t} + \{S_{dw}, H_0\} \right) \right]$$

$$+ \int dt \int d\zeta\, d\mu \int d\Lambda \left[-\left(\phi_{Moon} + \langle \phi_1 \rangle_\lambda\right)\{S_{dw}, f_0\} + \frac{f_0}{2}\left\langle \{\tilde{S}_1, \tilde{\phi}_1\} \right\rangle_\lambda \right]$$

$$- \frac{1}{4\pi G} \int dt \int d^2k\, \hat{\phi}_1(\boldsymbol{k}) \hat{\phi}_1^*(\boldsymbol{k}) \sqrt{k_x^2 + k_y^2}\,. \qquad (2)$$

Here, ϕ_{Moon} is the gravitational potential due to the perturbing satellite, ϕ_1 is the self-gravity of the rings, and $\tilde{\phi}_1$ is the λ-dependent part of this potential. S_{dw} is the λ-averaged part of the generating function associated with the density wave and $\tilde{S}_1$ is the λ-dependent part of the generating function associated with local gravitational instabilities. $\tilde{S}_1$ can be expressed in terms of the potential by inverting the equation,

$$\frac{\partial \tilde{S}_1}{\partial t} + \{\tilde{S}_1, H_0\} = \tilde{\phi}_1 \qquad (3)$$

Note that the gravitational energy for a razor thin disk is most conveniently expressed in Fourier space in the third line of eq. (2). As a result, the gravitational potential in the second line of eq. (2) must be expressed in Fourier space in order to perform variations with respect to the potential. This action principle is a powerful new formulation for density waves in planetary rings because it contains both the orbit-averaged model for density waves and the dynamics of local gravitational instabilities in one self-consistent model. If the S_{dw} terms are discarded, the remaining terms can be shown to generate Fuch's [8] integral equation for local gravitational instabilities in unperturbed particle disks. Conversely, if the ponderomotive potential term, $\left\langle\left\{\tilde{S}_1,\tilde{\phi}_1\right\}\right\rangle_\lambda$, is set equal to zero, the remaining terms in the action principle can be used to recover the usual Streamline Model for nonlinear density waves. Keeping all the terms results in a more complete version of Poisson's equation, which now includes the effects of local gravitational instabilities on the gravitational interaction between streamlines. In analogy to the plasma physics problem, the averaging transformation of the action principle has generated a non-trivial effective gravitational "dielectric" function for the rings. These effects are important in Saturn's rings where frequent collisions between ring particles damp the relative velocities such that the rings are close to being gravitational unstable. Another advantage of the variational principle is that it facilitates a straight-forward derivation of conservation laws via Noether's theorem [6]. In particular, the wave action conservation law that governs the flux of angular momentum carried by the density wave is readily derived by a simple variation with respect to the phase angle of the wave. The angular momentum flux carried by local gravitational instabilities has previously been neglected in the Streamline Model [3], [4] for density waves, but this flux is captured by the ponderomotive potential term in the action principle. This contribution to the angular momentum flux is likely to be significant in strongly perturbed regions of Saturn's rings.

3. Summary and Conclusions

The Streamline Model for density waves in planetary rings has been extended by replacing the usual orbit-averaging operation with Lie-transform perturbation theory in a Hamiltonian formulation of the problem. The resulting action principle facilitates modeling the self-consistent interactions between local self-gravity wakes and large-scale density waves for the first time. Since the Cassini spacecraft images of Saturn's rings give abundant observational evidence for kilometer-scale structures in strongly perturbed regions of the rings [1], [2], these new models are likely yield new insights to ring dynamics.

Acknowledgements

Glen Stewart was supported by a grant from NASA's Solar System Workings research program and the Cassini project.

References

[1] Porco, C. C., et al.: Cassini imaging science: Initial results on Saturn's rings and small satellites, *Science*, 307, pp. 1226-1236, 2005.

[2] Cuzzi, J. N., Filacchione, G., and Marouf, E. A.: The rings of Saturn. In *Planetary Ring Systems: Properties Structure, and Evolution*, M. S. Tiscareno and C. D. Murray, Eds., Cambridge Univ. Press, pp. 51-92, 2018.

[3] Shu, F. H., Yuan, C., and Lissauer, J. J.: Nonlinear spiral density waves: An invicid theory, *Ap. J.*, 291, pp. 356-376, 1985.

[4] Borderies, N., Goldreich, P. and Tremaine, S.: Nonlinear density waves in planetary rings, *Icarus*, 68, pp. 522-533, 1986.

[5] Brizard, A.: Eulerian action principles for linearized reduced dynamical equations, *Phys. Plasmas*, 1, pp. 2460-2472, 1994.

[6] Brizard, A.: Variational principles for nonlinear gyrokinetic Vlasov-Maxwell equations, *Phys. Plasmas*, 7, pp. 4816-4822, 2000.

[7] Brizard, A.: Perturbative variational formulation of Vlasov-Maxwell equations for linearized reduced dynamical equations, *Phys. Plasmas*, 25, 112112, 2018.

[8] Fuchs, B.: Density waves in the shearing sheet. I. Swing amplification, *Astron. & Astrophys.*, 368, pp. 107-121, 2001.