

## Orbital stability in the Solar System for all inclinations and eccentricities.

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### Abstract

Applying the technique of dynamical maps we study the orbital stability of test particles in the Solar System in the space  $(a, e, i)$  defined by  $0.1 < a < 38$  au,  $0 < e < 0.9$  and  $0^\circ < i < 180^\circ$  identifying the unstable and stable regions. We find stable niches where small bodies can survive even for very high eccentricities. Mean motion resonances (MMRs) play a fundamental role providing stability against the planetary perturbations specially for high inclination orbits. A stability band approximately near  $i \sim 150^\circ$  is present all along the Solar System. We find that the known population of objects with semimajor axes between 10 and 30 au is evolving inside a highly unstable region according to our maps. For the inner Solar System we find that the region between the Hildas and Jupiter is more stable for high eccentricity orbits than for low eccentricity ones.

### 1. Introduction

Since the work of Torbett [1] many studies have been done analyzing the stability of orbits in the plane  $(a, e)$  assuming in general small orbital inclinations providing a complete view of the dynamics for near ecliptic objects. However, considering the growing population of high inclination and even retrograde objects, it is necessary to extend these studies for the full range of inclinations and eccentricities.

### 2. Dynamical maps

We constructed dynamical maps integrating test particles perturbed by the actual system of planets from Venus to Neptune with initial conditions of the particles uniformly distributed in the interval  $0.1 < a < 38$  au,  $0^\circ < i < 180^\circ$  and with particular initial values for  $e$  from 0 to 0.9 in steps of 0.1. For the initial set  $(\omega, \Omega, M)$  we took random values between  $0^\circ$  and  $360^\circ$  from an uniform distribution. Each test particle was integrated for 1000 orbital periods with an

output of one particle's orbital period and we calculated the mean orbital elements after 100 orbital revolutions recording the maximum calculated differences  $\Delta a, \Delta e, \Delta i$  of the mean elements. Taking the mean instead of the osculating values we eliminate the short period variations and focus on the diffusion of the orbital elements in a timescale long enough to detect variations but short enough to avoid the loss of the memory of the initial conditions.

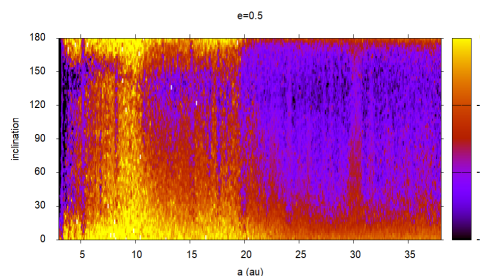


Figure 1: Diffusion in  $a$  for test particles with initial random  $\omega, \Omega, M$  and initial  $e = 0.5$ . The logarithmic colour scale indicate  $(\Delta a)/a = 1, 0.1, 0.01$  and  $0.001$ . The vertical bands are generated by MMRs.

### 3. Results

Figure 1 shows one of the maps for  $\Delta a/a$  in logarithmic colour scale. Black regions correspond to  $(\Delta a)/a \leq 0.001$  and yellow regions to  $(\Delta a)/a \geq 1$ . So, dark regions represent zones where diffusion in semimajor axis is minimum, which we can associate to stable regions, typical of secular evolution. Inside stable -dark- regions, MMRs appear as vertical -light- structures because they generate oscillations in the semimajor axes. But, inside unstable regions the MMRs appear as dark vertical lines because the planetary perturbations cannot break the small amplitude oscillations typical of resonant motion, specially for strong MMRs. These strong MMRs are clearly de-

fined in all panels and specially in the high eccentricity regime indicating that they provide some stability inside the chaotic regions. The resonances that persist in several panels are shown in the table. We have detected several exterior resonances of the type 1:N with Jupiter up to the resonance 1:15 at 31.6 au, confirming they are the strongest ones as showed by [2].

Table 1: Most common resonances detected.

Resonance	$a$ (au)
1:1V	0.72
1:1E	1.00
4:1J	2.06
3:1J	2.50
2:1J	3.28
3:2J	3.97
4:3J	4.29
1:1J	5.2
2:3J	6.8
1:2J	8.2
1:3J	10.8
1:4J	13.1
1:5J	15.2
1:2S	15.2

Our results show that the preferences for captures in retrograde resonances obtained by [3] are explained by the lower magnitude of the planetary perturbations on retrograde orbits.

A horizontal stable band is clearly defined by  $i \sim 150^\circ \pm 20^\circ$  indicating that orbits with these inclinations are the least perturbed orbits for a wide range of eccentricities and semimajor axes. The observed population of retrograde objects shows a concentration in this stable band.

The known population of objects with  $10 < a < 30$  au (around 300 Centaurs plus 70 comets) is mostly evolving in unstable regions. Another interesting result of this study is that for orbits located between the Hildas and Jupiter the high eccentricity orbits are more stable than the lower eccentricity ones.

## Acknowledgements

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## References

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