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To analytical theory of Mercury rotation: planetary perturbations

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Introduction. Now the new and wide opportunities for the accurate determination and description of the Mercury gravitational field and its rotation are opened in the Messenger mission to the planet. Therefore theoretical estimations of expected effects in rotation of Mercury from influence of the liquid core represent the importa nt interest [1], [2]. In the work the analytical theory of rotational motion of Mercury which is considered as a celestial body with a liquid core and rigid non-spherical mantle is developed. We consider orbital motion of Mercury on elliptical precessing orbit and in more general treatment of problem we take into account planetary perturbations of forced orbital motion of planet. For the basic variables: Andoyer, Poincare and Eulerian angles, and also for various dynamic characteristics of Mercury the tables for amplitudes, periods and phases of perturbations of the first order have been constructed. Resonant periods of free librations of given model of Mercury have been estimated. The influence of a liquid core results in decreasing of the period of free librations in longitude approximately on 30%, and in change of the period of free pole wobble of Mercury on 50%. In the first approximation the liquid core does not render influence on the value of Cassini's inclination and on the period of precession of the angular momentum vector. However it causes an additional "quasi-diurnal" librations with period about 27.165 days. In comparison with model of rigid non-spherical of Mercury the presence of a liquid core should result in remarkable increase of amplitudes of Mercury librations in longitude on 50 %. The theory has been constructed by the full taking into account planetary perturbations in orbital motion of Mercury. For this purpose the spesial trigonometric developments of the sperical functions of spherical coordinates of Mercury and full force function of the problem have been constructed. Cassini's laws are formulated in first in given exect treatment of the problem as generating periodic solution for intermediate conditionally-periodic solution of the problem. The main perturbations in librations are analized.

1 Development of analytical theory of rotational motion of Mercury with liquid core and rigid mantle. The work has been realized in following stages. 1. Canonical equations of rotation of Mercury with liquid core and elastic mantle in Andoyer and Poincare variables have been constructed. Developments of second harmonic of force function of Mercury in pointed variables have been obtained for accurate trigonometric presentation of planetary perturbations of Mercury orbital motion. 2. The main equation for determination of Cassini's inclination and its solution has been obtained in the case of accurate orbit of Mercury. An dynamical explanation of Cassini's laws has been done for model of Mercury with liquid core [1]. 3. Compact formulae for perturbations of the first order have been constructed for general used variables and for different kinematical and dynamical characteristics of Mercury. 4. Analytical formulae for 4 periods of free librations of Mercury have been constructed: for librations in longitude, in pole wobble, for free precession, and "quasi-diurnal" librations, caused by the liquid core. 5. Planetary orbital perturbations in Mercury rotation have been studied.

2 Development of force function. In first we have obtained trigonometric development of second harmonic of gravitational potential of Mercury in following form:

$$U=\sum_{\nu}C_{\nu}^{0}\cos\Theta_{\nu}+\sum_{\varepsilon=\pm1}\sum_{\nu}C_{\nu}^{(1)}\cos\left(g-\varepsilon\Theta_{\nu}\right)+$$
$$+\sum_{\varepsilon=\pm1}\sum_{\nu}C_{\nu}^{(2)}\cos\left(g-2\varepsilon\Theta_{\nu}\right)+\sum_{\nu}S_{\nu}^{(0)}\sin\Theta_{\nu}+$$
$$+\sum_{\nu}S_{\nu}^{(0)}\sin\Theta_{\nu}+\sum_{\varepsilon=\pm1}\sum_{\nu}S_{\nu}^{(1)}\sin\left(g-\varepsilon\Theta_{\nu}\right)+$$
$$+\sum_{\varepsilon=\pm1}\sum_{\nu}S_{\nu}^{(2)}\sin\left(g-2\varepsilon\Theta_{\nu}\right)\cdot \qquad (1)$$

EPSC Abstracts,
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Where arguments $\Theta_{\mathbf{v}} = \nu_1 L_{Me} + \nu_2 L_V + \nu_3 L_E + + \nu_4 L_{Ma} + + \nu_5 L_{Ju} + \nu_6 L_{Sa} + \nu_7 L_{Ur} + \nu_8 L_{Ne}$ are linear combination of mean longitudes of orbital motions of 8 planetas: Mercury (L_{Me}), Venus (L_V), the Earth (L_E), Mars (L_{Ma}), Jupiter (L_{Ju}), Saturn (L_{Sa}), Uran (L_{Ur}), Neptune (L_{Ne}); $\mathbf{v} = (\nu_1, \nu_2, \nu_3, ..., \nu_8)$ are integer indexes. Coefficients C_ν^j and S_ν^j ($j = 0,1,2$) are known functions of angle of inclination of angular momentum vector with respect to polar axis of inertia ρ , Andoyer variables θ and l , determining orientation of polar axis of inertia of Mercury with respect to intermediate reference system connected with angular momentum. Also these coefficients depend from the dynamical oblatenesses or from coefficients of second harmonic of Mercury gravitational potential J_2 / I and C_{22} / I . I is here dimensionless moment of inertia of Mercury. Coefficients C_ν^j and S_ν^j ($j = 0,1,2$) depend also from the coefficients of developments of spherical functions of spherical orbital heliocentric coordinates of Mercury in trigonometric series on arguments $\Theta_{\mathbf{v}}$. Generally specified coefficients are not constants, and contain additives of the first and second degree on time which take into account the contribution of secular orbital planetary perturbations the force function.

The coefficients of perturbations of the first order in Mercury rotation also have structure similar to (1). These coefficients were constructed on the base special method of construction of conditionally-periodic solutions of Hamiltonian systems in resonant case [3].

3 Motion on Cassini's - Colombo. According to a developed method at the first stage the equations of rotary motion average in view of a resonance are analyzed. The stationary solution of the average equations which corresponds to motion of Mercury under Cassini's-Colombo laws is found. For this solution ascending nodes of an orbit and equator of Mercury on the Laplace plane coincide. And the angle between a vector of the angular moment and a polar axis of inertia of Mercury is determined in dependence on parameters of the perturbed orbit and from dynamical oblatenesses of a planet and makes: $\rho = \rho_0 = 1'30"$.

4 Structure of perturbations of the first order and their tabulation. For example, periodic perturbations in inclination ρ and in node h of angular momentum of Mercury are determined by formulae:

$$\rho = \rho_0 + \sum_{\nu} \rho_\nu^{(1)} \cos\Theta_\nu + \sum_{\nu} \bar{\rho}_\nu^{(1)} \sin\Theta_\nu ,$$

$$h = \pi + \sum_{\|\nu\|} h_\nu^{(1)} \sin\Theta_\nu + \bar{h}_\nu^{(1)} \cos\Theta_\nu . \qquad (2)$$

Here $\rho_0 = 1'30"$ is the Cassini's inclination of the Moon; $\bar{\rho}_\nu^{(1)}$, $h_\nu^{(1)}$ are constant coefficients.

An influence of the liquid core and its ellipticity is determined by positive correction to amplitudes of librations for model of the rigid Mercury. If the amplitudes of librations of rigid Moon we note as 1, so the corresponding amplitudes of librations of the Mercury with a liquid core will be characterized by parameter $1+L$, where correction for liquid core is determined by formula $L = C_c\left(1-\varepsilon^2\right)/C \approx C_c/C = 0.5$, where C and C_c is the polar moments of inertia of the Moon and its core; $\varepsilon = \left(a^2 - b^2\right) / \left(a^2 + b^2\right) \approx (a-b)/a$ is an ellipticity of equatorial ellipse of core cavity with semi-axes a and b . So all amplitudes of librations in longitude due to the liquid core are increased on 50%. An effect of ellipticity has more smaller order. The perturbations of the first order for others variables and considered dynamical characteristics have the structure similar to the formula for variables (2).

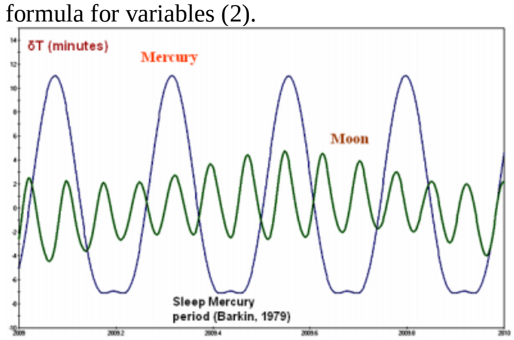

Fig. 1. Variations of periods of axial rotation of the Moon and Mercury in 2009 year.

On Fig.1 for comparison the variations of periods of diurnal rotations of the Moon and Mercury

EPSC Abstracts,
Vol. 4, EPSC2009-136, 2009
European Planetary Science Congress,

(δT) are given for 2009 year. Variations are remarcable and given in minutes (ordinate axis). It is pointed a sleep period of Mercury rotation [4], for which the angular velocity almoust constant for comparetively long period of time at passage of pericenter. The preliminary analysis of tabulation of planetary perturbations in Mercury axial rotation, in its pole motion and in inclination and precession of its polar axis of inertia has been fulfilled.

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