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Effects of second approximation of the figure theory for Jupiter's satellite Io: 1. Theory and numerical solution

V.N.Zharkov and **T.V.Gudkova**
Schmidt Institute of Physics of the Earth, Russian Academy of Sciences, Russia (gudkova@ifz.ru/ Fax: +7-499-2556040)

Abstract

Integro-differential equations of the theory of figure to second order have been first solved numerically. The corrections of second order in smallness to the gravitational moments J_2 and C_{22} decrease the third decimal digit of model gravitational moments by two units.

1. Introduction

All Galilean satellites are in synchronous rotation; their orbits are nearly circular and lie in the equatorial plane of Jupiter. Io is the large satellite closest to Jupiter. Therefore, the influence of Jupiter's tidal potential on the equilibrium figure and gravitational field of Io is appreciably stronger than it is on the remaining large satellites. For the theory of Io's figure to be consistent with currently available observational data, it must include effects of the second order in smallness. In the first approximation, the ratio of the moments J_2 and C_{22}: $J_2=10/3C_{22}$. The parameters of the gravitational field for the Galilean satellites determined in the Galileo space mission have shown that relation holds with a high accuracy. Consequently, these bodies have equilibrium figures. For Io, J_2 and C_{22} are given to the fourth decimal place. This main relation that is used to judge whether Io has an equilibrium figure was derived in the first approximation. Therefore, the following question arises: With what accuracy is this theoretical ratio valid? To answer this question, we must construct a theory in the next (second) approximation by including the terms of order α^2 (α is the small parameter of theory of figure, defined as $\alpha=3\pi/(G\rho_0\tau^2)$, where ρ_0 and τ are the average density and rotation period of Io, respectively. This is the main goal of our analysis.

2. Results

The method for solving the equations of the figure theory is described in detail in [1]. A computer code was developed to calculate figures functions s_4(s), s_{42}(s) and s_{44}(s) for the Io models. Tables 1 and 2 give results of numerical solutions of figure equations for the Io1 and Io3 models [2] at s=s_1 (for visual perception tables list the normalized figure parameters $\bar{s}_2$, $\bar{s}_{22}$, $\bar{s}_{31}$, $\bar{s}_{33}$, $\bar{s}_4$, $\bar{s}_{42}$ and $\bar{s}_{44}$ and gravitational moments $\bar{J}_2$, $\bar{C}_{22}$ $\bar{C}_{31}$, $\bar{C}_{33}$, $\bar{J}_4$, $\bar{C}_{42}$ and $\bar{C}_{44}$). As seen from Table 1, including the second order terms in $\bar{J}_2$ and $\bar{C}_{22}$ decreases the values by two units in the third decimal digit. To make clear insight into the problem, we plot figure functions $\bar{s}_2(s)$, $\bar{s}_{22}(s)$ (Fig. 1a) and $\bar{s}_4(s)$, $\bar{s}_{42}(s)$, $\bar{s}_{44}(s)$ (Fig. 1b).

Table 1: Normalized figure parameters $\bar{s}_n$ and gravitational coefficients $\bar{J}_n$ and $\bar{C}_{nm}$ (the first order) and the second order corrections for the model Io1.

Parameters	Model values
$-\bar{s}_2$, 10^{-3}	1.4709
$\bar{s}_{22}$, 10^{-3}	1.5287
$-\bar{J}_2$, 10^{-6}	832.25
$\bar{C}_{22}$, 10^{-6}	864.90
$-\Delta\bar{s}_2$, 10^{-3}	0.0017
$\Delta\bar{s}_{22}$, 10^{-3}	0.0022
$-\Delta\bar{J}_2$, 10^{-6}	1.6993
$\Delta\bar{C}_{22}$, 10^{-6}	2.2378

Figure functions $\bar{s}_2(s)$, $\bar{s}_{22}(s)$ and $\bar{s}_{31}(s)$, $\bar{s}_{33}(s)$ are proportional to the Love function $h_2(s)$ and $h_3(s)$, respectively. The outer regions of Io's interiors influence more the Love number $h_3(s)$, then $h_2(s)$. The same fact is seen, when comparing the figure function $\bar{s}_2(s)$, $\bar{s}_{22}(s)$ (Fig. 1a) and figure functions $\bar{s}_4(s)$, $\bar{s}_{42}(s)$, $\bar{s}_{44}(s)$ (Fig. 1b). In that way, functions of the second approximation $\bar{s}_4(s), \bar{s}_{42}(s), \bar{s}_{44}(s)$ sound the density distribution of the external zones to a greater extent than functions of the first approximation $\bar{s}_2(s)$, $\bar{s}_{22}(s)$.

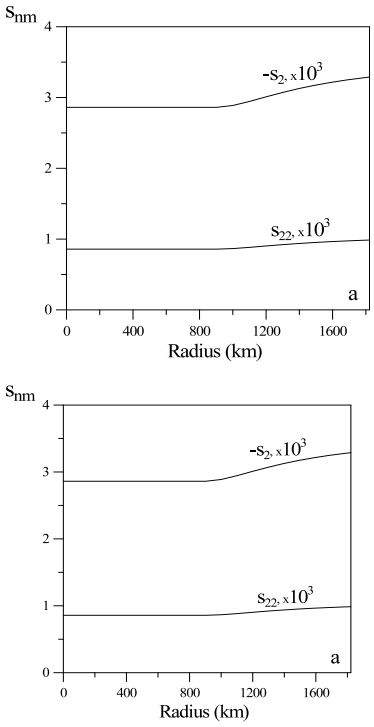

Figure 1: The distribution of the parameters of the equilibrium figure of Io $s_2(s)$, $s_{22}(s)$ (a) and $s_4(s)$, $s_{42}(s)$, $s_{44}(s)$ (b) along the planetary radius. The functions $s_2(s)$, $s_{22}(s)$ and $s_4(s)$, $s_{42}(s)$, $s_{44}(s)$ are not normalized.

Table 2: Normalized figure parameters $\bar{s}_n$ and gravitational coefficients $\bar{J}_n$ and $\bar{C}_{nm}$ (the second order) for the models Io1 and Io3

Parameters	Io1	Io2
$-\bar{s}_{31}$, 10^{-6}	2.8404	2.8334
$\bar{s}_{33}$, 10^{-6}	3.6670	3.6579
$\bar{s}_4$, 10^{-6}	3.4666	3.4498
$-\bar{s}_{42}$, 10^{-6}	3.9415	3.9223
$\bar{s}_{44}$, 10^{-6}	2.2472	2.2190
$-\bar{C}_{31}$, 10^{-6}	1.1266	1.1196
$\bar{C}_{33}$, 10^{-6}	1.4545	1.4454
$-\bar{J}_4$, 10^{-6}	3.0926	3.0952
$-\bar{C}_{42}$, 10^{-6}	3.5163	3.5014
$\bar{C}_{44}$, 10^{-6}	1.9096	1.9008

3. Conclusion

The effects of the second approximation on the figure parameters and gravitational moments of the satellite Io have been considered. It turns out that the account of the second order values decrease gravitational moments J_2 and C_{22} by 2 units in the third decimal digit. To calculate the figure parameters s_4, s_{42} and s_{44} and consequently gravitational moments J_4, C_{42} and C_{44}, three integro-differential equations [3] for trial model density distributions were first solved numerically and, as a result, all second order corrections were obtained.

Acknowledgements

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References

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