



The obliquity of Titan as evidence for the presence of a liquid ocean beneath the surface and perspectives for the determination of the ice shell thickness of Europa.

R.-M. Baland, T. Van Hoolst, M. Yseboodt and Ö. Karatekin
 Royal Observatory of Belgium, Avenue Circulaire 3, B-1180 Bruxelles, Belgium, (Rose-Marie.Baland@oma.be)

1. Introduction

A value of the obliquity of Titan, $\varepsilon \simeq 0.3^\circ$, has been obtained from a study of Cassini radar images [1]. From radio tracking, the quadrupole field of Titan has been found to be consistent with a body in hydrostatic equilibrium, and the resulting moment of inertia, C , is about $0.34MR^2$ [2]. The obliquity value is, however, not consistent with the moment of inertia because the 0.3° obliquity value implies $C = 0.55MR^2$, according to a study of the Cassini state generalised to a multi-frequency node precession for Titan [3], in which the ice outer shell is fully coupled with the interior. With a liquid water ocean beneath the ice shell, the shell is only partially coupled with the interior by gravitational, pressure and viscous torques, which may result in a different obliquity value.

Here, by considering the gravitational and pressure torques between the different layers of the satellite, as in a study of the librations of the Galilean satellites [4], we show that the obliquity value of the exact Cassini state (Titan is close to the exact Cassini state, see [1]) can be consistent with the moment of inertia. We also apply the same method to Europa, for which the moment of inertia is known, in order to estimate its obliquity and the thickness of the ice shell. The viscous torques have been neglected because the precession period of Titan and Europa (about 600 and 30 years, respectively) are smaller than the timescale at which the viscosity of a subsurface ocean is effective (about 30000 years).

2. Obliquity model

In a first approximation, Titan and Europa can be considered as synchronous bodies (n is the orbital and rotational rate), with a constant orbital precession rate $\dot{\Omega}$ and a constant orbital inclination i , locked in the (exact) Cassini state.

If the satellite is considered as rigid, the obliquity ε of the Cassini state is related to the polar moment of inertia C by

$$\sin(i + \varepsilon) = \frac{\frac{3}{2}MR^2 \frac{J_2 + 2C_{22}}{C} n}{\frac{3}{2}MR^2 \frac{J_2 + 2C_{22}}{C} n + \dot{\Omega}} \sin i \quad (1)$$

For Titan, with $\varepsilon = 0.3^\circ$, $C_{20} = -31.808 \cdot 10^{-6}$, $C_{22} = 9.983 \cdot 10^{-6}$, it follows that C must be equal to $0.5MR^2$ (close to the generalised Cassini state value of [3]) which is physically impossible and is not consistent with the value derived from the quadrupole field estimation in [2].

If the satellite is considered to have a liquid water ocean beneath an ice shell and above a solid interior, and by considering the gravitational and pressure coupling between the three layers, the obliquity of the Cassini state for the shell and the interior, ε_s and ε_i , are the solutions of

$$\sin(i + \varepsilon_s) = \frac{(f - \kappa_s(\kappa_i + C_i \dot{\Omega})n)}{g} \sin i \quad (2)$$

$$\sin(i + \varepsilon_i) = \frac{(f - \kappa_i(\kappa_s + C_s \dot{\Omega})n)}{g} \sin i \quad (3)$$

with

$$f = K(\kappa_i + \kappa_s) \cos(\varepsilon_i - \varepsilon_s) \quad (4)$$

$$g = K(\kappa_i + \kappa_s + C_s \dot{\Omega} + C_i \dot{\Omega}) \cos(\varepsilon_i - \varepsilon_s) - (\kappa_i + C_i \dot{\Omega})(\kappa_s + C_s \dot{\Omega})n \quad (5)$$

Here, C_s and C_i are the moments of inertia of the shell and of the interior. K represents the strength of the internal gravitational torque between the shell and the interior and κ_s and κ_i represent the external torques of the central planet on the shell and on the interior, respectively. These internal and external torques also include the effect of the pressure in the middle liquid layer on the shell and on the interior [4].

3. Obliquity versus moment of inertia for Titan

For a representative range of hydrostatic interior structure models constrained by the mass and the radius, we compute the moment of inertia C and the obliquity of the shell ε_s . The obliquity versus the moment of inertia is given in Fig. 1.

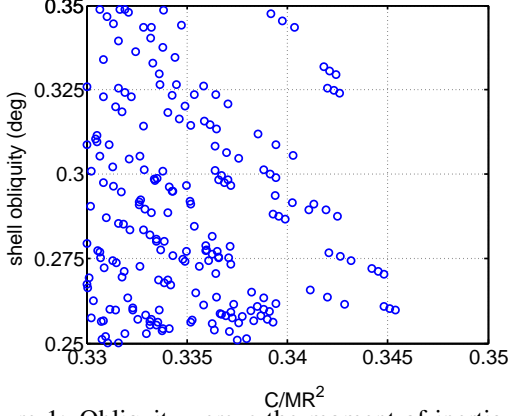


Figure 1: Obliquity versus the moment of inertia for a range of interior structure models constrained by the mass and radius of Titan.

We find that the measured value of the obliquity $\varepsilon \simeq 0.3^\circ$ is in a good agreement, for several internal structure models, with the value of the moment of inertia of about $0.34MR^2$ resulting from the quadrupole field. Since the relation between ε and C is much better described by including a subsurface ocean, our results suggest that Titan probably has a liquid layer beneath its ice surface.

We also studied whether the obliquity can be used to determine the thickness of the ice shell, as can be done for the librations (see e.g. [4]). We considered another range of interior structure models constrained by the mass, the radius and $C = 0.34MR^2$ but we did not see any relation between the obliquity and the ice shell thickness. Therefore the measurement of the obliquity will not help to constrain the ice shell thickness of Titan.

4. Application to Europa

For Europa, the moment of inertia has been estimated to be $C = 0.346MR^2$ [5]. The obliquity has not been measured yet. For interior structure models constrained by the mass, the radius and the moment of inertia, the shell obliquity versus the ice shell

thickness is calculated from Eqs (2) and (3) and is given in Fig.2. The obliquity is lower than its value in the rigid case (0.055°). We clearly see a linear relation between the obliquity and the shell thickness. The obliquity depends weakly on the radius and the density of the other layers. Therefore a measurement of the obliquity could constrain the ice shell thickness of Europa.

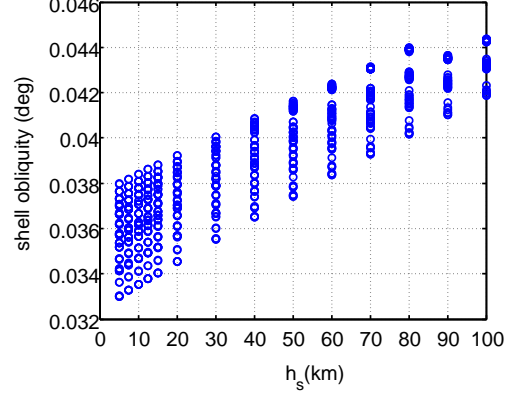


Figure 2: Obliquity versus the ice shell thickness for a range of interior structure models constrained by the mass and the moment of inertia of Europa.

The ice shell thickness of Titan cannot be constrained by the obliquity because a resonance between a free mode and the precession rate amplifies the obliquity in such a way that the obliquity depends on all the the interior parameters structure (radius and density of the different layers). For Europa, such a resonance does not exist.

References

- [1] Stiles B.W. et al.: Determining Titan's spin state from Cassini RADAR images, *The Astronomical Journal*, 135, 1669–1680, 2008.
- [2] Iess L. et al.: Gravity field, shape, and moment of inertia of Titan, *Science*, 327, 1367, 2010.
- [3] Bills. B. and Nimmo F. Spin, gravity, and moments of inertia of Titan, EPSC2009-553.
- [4] Baland R.-M. and Van Hoolst T., Librations of the Galilean satellites: the influence of global internal liquid layers, *Icarus* (in press).
- [5] Anderson J.D. et al. (1998) Anderson, J. D. et al.: Europa's Differentiated Internal Structure: Inferences from Four Galileo Encounters, *Science*, 281, 2019–2022, 1999.