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Long-period forced librations in longitude of Mercury and dissipation

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Abstract

We investigate the long-period forced librations in longitude of Mercury due to the planetary perturbations on Mercury's orbit and the link with the interior and dissipation parameters. Particularly, the phase shift of these librations is a function of the moments of inertia and the dissipation (Peale et al, 2009, Yseboodt et al, 2010).

1. Introduction

Mercury is in a special resonant state in which two revolution periods are equal to three rotation periods. This resonant state leads to librations in longitude whose periods are harmonics of the orbital period (88 days). These librations are due to the gravitational torque of the Sun on the permanent flattening of the planet. The amplitude of the 88-day forced libration is proportional to the moment of inertia ratio $(B-A)/C_m$ where $A < B$ are the moments of inertia of the planet and C_m is the moment of inertia of the silicate shell about the spin axis. The phase of these librations is 0 with respect to the solar anomaly.

Additionally to these main librations, Mercury also experiences planetary forced librations, due to the perturbations on the orbital motion of Mercury induced by the other planets (Peale et al 2007, 2009, Dufey et al 2008). Therefore the associated periods are related to the orbital motion of the biggest planet (Jupiter) or to planets close to Mercury (Venus or the Earth).

An analytical formulation for the long-period forced librations of Mercury has been given in Yseboodt et al (2010). This model is a generalization of the harmonic oscillator model with a forcing due to the planets. It also gives the link between the phase shift and the moments of inertia and the dissipation parameters. Is it possible to find information about the interior of Mercury from a dissipation shift of the long-period forced librations in longitude?

2. The equations

The planetary part of the libration angle $\psi_m(t)$ is:

$$\psi_m(t) = \sum_i \psi_i \cos(w_i t + \phi_{\psi_i}) \tag{1}$$

where w_i is the planetary frequency, the amplitude of the forced libration

$$\psi_i = \sqrt{\gamma_i^2 + \lambda_i^2 + 2\gamma_i \lambda_i \cos \phi_i^R}\,. \tag{2}$$

and its phase ϕ_{ψ_i} is:

$$\tan \phi_{\psi_i} = \frac{\gamma_i \sin(\phi_{\lambda_i} + \phi_i^R) + \lambda_i \sin \phi_{\lambda_i}}{\gamma_i \cos(\phi_{\lambda_i} + \phi_i^R) + \lambda_i \cos \phi_{\lambda_i}}\,. \tag{3}$$

We need to define of γ_i and ϕ_i^R:

$$\gamma_i = \lambda_i\, w_i^2 / \sqrt{(w_0^2 - w_i^2)^2 + w_i^2 b^2}\,. \tag{4}$$

$$\begin{aligned} \sin \phi_i^R &= -w_i\, b / \sqrt{(w_0^2 - w_i^2)^2 + w_i^2 b^2} \\ \cos \phi_i^R &= (w_0^2 - w_i^2) / \sqrt{(w_0^2 - w_i^2)^2 + w_i^2 b^2} \end{aligned} \tag{5}$$

λ_i and ϕ_{λ_i} are the known amplitude and the phase of the planetary perturbation of Mercury's orbit, w_0 is the free libration frequency. The dissipation parameter b is defined as $b = F/(C_m n) + k/C_m$. k is a constant coupling the core and the mantle, F is a constant depending on the mantle properties and shape, n is the mean motion.

3. The equations if the dissipation can be neglected

If the dissipation is neglected, the phase shift ϕ_i^R (Eq. 5) is 0 for the frequencies smaller than the free libration frequency and is 180° for the frequencies above w_0.

The phase $\phi_{\psi_i} \approx \phi_{\lambda_i} - \phi_{i,b=0}^R \approx \phi_{\lambda_i} + \phi_{i,b=0}^R$ (Eq 3) is reduced to ϕ_{λ_i} when $w_0 > w_i$ and to $\phi_{\lambda_i} + 180°$ when $w_0 < w_i$.

When $w_0 < w_i$ and the frequency w_i is far from

the resonance frequency w_0, since b is small with respect of w_i, the coefficient γ_i (Eq. 4) is reduced to $\gamma_i \approx \lambda_i \left(1 + \frac{w_0^2}{w_i^2}\right)$.
When $w_0 > w_i$ and if the frequency w_i is far from the resonance frequency w_0, the coefficient γ_i is $\gamma_i \approx \frac{\lambda_i}{w_0^4}(w_i^4 + w_0^2 w_i^2)$.

4. Estimation of the dissipation parameter b

Using a reasonable damping with ν = 0.01 cm^2/s (the kinematic viscosity of the core, see Peale (2005), eq 38 and following) and $k_2/Q = 0.004$, the damping parameter is $b_{\text{nominal}} = 10^{-5}$/y. This corresponds to a damping time E/b of about 270 000 years (estimation of Peale, 2005).
A large value for the dissipation parameter b_{HD} is 5×10^{-4}/y (ν = 30 cm^2/s and $k_2/Q = 0.04$). This corresponds to a very short damping time of about 5 000 years.

5. The effect of the dissipation on the forced librations

For a particular frequency, from Eq 1, the forced libration due to the planetary perturbations is

$$\begin{aligned} \psi_m(t) &= (A^{b=0} + \Delta A_{\text{diss}}) \cos(w\,t + \phi_\psi^{b=0} + \phi_{\text{diss}}) \\ &\approx A^{b=0} \cos(w\,t + \phi_\psi^{b=0}) + \Delta A_{\text{diss}} \cos(w\,t + \phi_\psi^{b=0}) \\ &\quad + A^{b=0} \sin \phi_{\text{diss}} \sin(w\,t + \phi_\psi^{b=0}) \end{aligned} \tag{6}$$

where the small phase shift ϕ_{diss} due to the dissipation is defined as $\phi_{\text{diss}} = \phi_\psi - \phi_\psi^{b=0}$.

Period (y)	$A^{b=0}$ (as)	$\Delta A_{\text{diss HD}}$ (as)	$\phi^R_{b=0}$ (deg)	ϕ_{diss} (deg)	$\phi_{\text{diss HD}}$ (deg)	$A \sin \phi_{\text{d}}$ (as)	$A \sin \phi_{\text{d HD}}$ (as)
5.7	3.63	0	180.	0	0.15	0	0.01
5.9	1.39	0	180.	0	0.15	0	0
11.9	56.73	-0.04	180.	0.05	2.3	0.05	2.28
6.6	0.58	0	180.	0	0.14	0	0
14.7	1.56	0	0	0	-0.09	0	0

Table 1: Amplitudes and phases of the planetary librations and effet of the dissipation for $(B-A)/C_m = 2.05\,10^{-4}$. The dissipation parameter b is nominal ($b = 10^{-5}$/y) or very large ($b = 5\,10^{-4}$/y, columns labeled with HD).

As we can see on Table 1, the uncertainty related to the dissipation affects very slightly the libration amplitudes (less than 0.05 as).
The only libration phase ϕ_{diss} affected by the dissipation is the 11.86y libration. If $b = 10^{-5}$/y (nominal value for the damping in about 270 000 years), there is a shift of $0.04°$ or 0.55 days for the 11.86y libration.
If the dissipation is large ($b = 5\,10^{-4}$/y), there is a shift of 2.3° or 28 days for the 11.86y libration.
Very close to the resonance, a change in the dissipation will affect more the phase than the amplitude of the 11.86 year libration. Figure 1 shows the phase shift due to the dissipation for the 11.86y libration as a function of $(B-A)/C_m$ and the dissipation parameter b. Margot et al (2007) have a best fit value of $(B-A)/C_m$ of 2.03 10^{-4}.

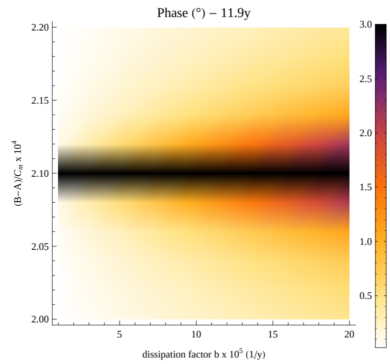

Figure 1: Phase ϕ_{diss} of the 11.86y libration in degrees as a function of $(B-A)/C_m$ and the dissipation parameter b.

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