



# Tidal dissipation and dynamical evolution of the Earth-Moon system: Revisited

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## Resume

Tidal dissipation and dynamical evolution of Earth Moon system is once again analyzed. Following Lambeck's approach with correction, a preliminary numerical solution set is acquired for the future and the past dynamical state of the Earth Moon system.

## 1. Introduction

Earth's spin rotation is slowing down and the Moon is receding from the Earth due to the tidal dissipation, as confirmed by Lunar Laser Ranging. Ocean bottom friction and tidal energy dissipation in the Earth's crust and mantle are associated together. The total effect may be represented by equivalent tidal phase lag. Other methods, which can estimate this rate, are (1) ocean tide modelling plus body tide dissipation modelling, (2) satellite orbit perturbation analysis. Also there exist certain fossil evidences, which keep records about rotational state of Earth Moon system during the Paleozoic or Pre-Cambrian Era. Extrapolation of the dynamical state of the Earth Moon system have been attempted by G. Darwin, H. Gerstenkorn, P. Goldreich and others [1 - 4]. This extrapolation is attempted in this study, with a few advances and new concepts.

## 2. Formulation

Using Kaula's former work about theory of satellite orbit perturbation [5], Lambeck developed formulas for change rate of semimajor axis, eccentricity, and inclination of satellite due to the tidal deformation of the planet induced by the satellite itself [6].

$$\begin{aligned}\frac{da}{dt} &= 2K_{lm} [F_{lmp}(i)]^2 [G_{lpq}(e)]^2 (l-2p+q) \sin \varepsilon_{lmpq} \\ \frac{de}{dt} &= K_{lm} \frac{\sqrt{1-e^2}}{ae} [F_{lmp}(i)]^2 [G_{lpq}(e)]^2 \\ &\quad \times [\sqrt{1-e^2} (l-2p+q) - l + 2p] \sin \varepsilon_{lmpq} \\ \frac{di}{dt} &= K_{lm} \frac{(l-2p) \cos i - m}{a\sqrt{1-e^2} \sin i} [F_{lmp}(i)]^2 [G_{lpq}(e)]^2 \\ &\quad \times [\sqrt{1-e^2} (l-2p+q) - l + 2p] \sin \varepsilon_{lmpq}\end{aligned}$$

where  $F_{lmp}(i)$  and  $G_{lpq}(e)$  are defined as by Kaula, and  $K_{lm}$  and  $\varepsilon_{lmpq}$  are defined as follows.

$$\begin{aligned}K_{lm} &= \frac{Gmk_l}{\sqrt{G(M+m)a}} \left(\frac{R}{a}\right)^{2l+1} \frac{(l-m)!}{(l+m)!} (2-\delta_{0m}) \\ \varepsilon_{lmpq} &= (l-2p)\omega + (l-2p+q)M^* + m(\Phi - \theta)\end{aligned}$$

Lambeck's estimates of equivalent phase angles for the whole effect of body tide and ocean tide are adopted for this study (e.g. 6.4, 6.6, 1.7, and 1.8 for M2, N2, O1, and K1 tide: [unit: degree]). But it should be noted that the third formula can not be used for the Moon, because the mass of the Moon is large (1/81 of the Earth's mass) and the angular momentum associated with the Moon's orbital motion is quite large - even larger than the Earth's spin angular momentum at present stage. So the third equation (Lunar inclination to the equator) must be abandoned and be replaced with another one based on consideration of the angular momentum transfer as described in Fig. 1.

## 3. Result and Discussion

Using the method described above, the dynamical state of the Earth Moon system in the future of 5

billion years is acquired and shown in Fig. 2. Also the past state of the Earth Moon system in the past is shown in Fig. 3. The calculations here are based on the assumption of constant equivalent phase angle in the whole lunar tidal bulge effect of the Earth, which is accurate at present state but may differ in either of future and past states. Paleontological evidences can constraint the value of equivalent phase angle. Another enhancement would be attained, if Earth Moon system angular momentum leakage due to solar tide is considered as well. More calculations incorporating these points will be done and be reported.

## 4. Figures

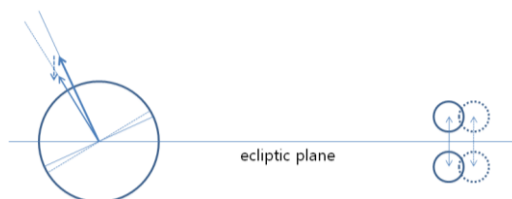


Figure 1: Schematic figure of the Earth Moon system. The Earth's spin angular momentum decrease is accompanied with recession of the Moon, increase of the Earth's obliquity, and decrease in the lunar inclination to the ecliptic.

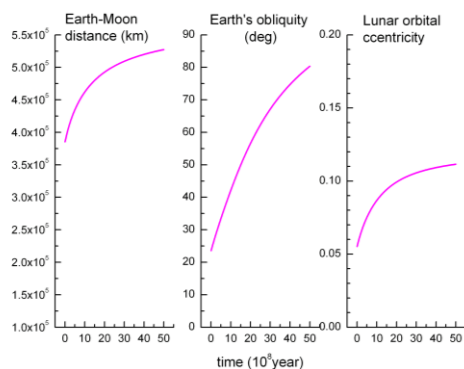


Fig. 2: State of the Earth Moon system in the future of 5 billion years (assuming constant phase angle)

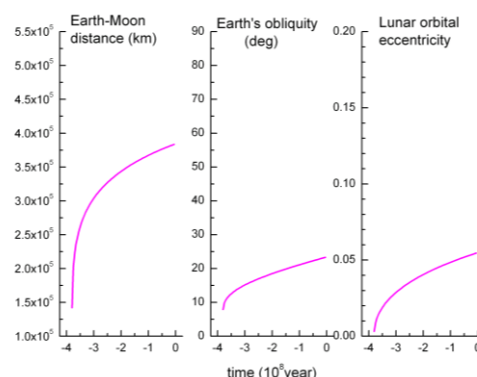


Fig. 3: State of the Earth Moon system in the past (assuming constant phase angle)

## References

- [1] Brosche, P. and Sundermann, J.(ed): Tidal Friction and the Earth's Rotation, Springer-Verlag N.Y., 345pp.,1982.
- [2] Lambeck, K.: Tidal dissipation in the ocean: astronomical, geophysical, and oceanographic consequences, Phil. Trans. R. Soc. Lond. A287, 545-594, 1977.
- [3] Goldreich, P.: History of Lunar Orbit, Rev. Geophys., 4, 411-439, 1966.
- [4] S. Na and Moon, W.: Tidal dissipation and dynamical evolution of the Earth Moon system: A review, 1984 annual report of the Centre for PreCambrian Studies, Univ. of Manitoba, 44-55, 1985
- [5] Kaula, W: Introduction to Satellite Geodesy, Blaisdell, 124pp., 1966.
- [6] Lambeck, K: The Earth's Variable Rotation: Geophysical Causes and Consequences, Cambridge Univ. Press, 449pp., 1980.

